CHAPTER 20
Vector Algebra and Three Dimensional Geometry

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. Let $\vec{A}$, $\vec{B}$, $\vec{C}$ be vectors of length 3, 4, 5 respectively. Let $\vec{A}$ be perpendicular to $\vec{B}+\vec{C}$, $\vec{B}$ to $\vec{C}+\vec{A}$ and $\vec{C}$ to $\vec{A}+\vec{B}$. Then the length of vector $\vec{A}+\vec{B}+\vec{C}$ is .... *(1981 - 2 Marks)*

2. The unit vector perpendicular to the plane determined by $P(1,-1,2)$, $Q(2,0,-1)$ and $R(0,2,1)$ is .... *(1983 - 1 Mark)*

3. The area of the triangle whose vertices are $A(1,-1,2)$, $B(2,1,-1)$, $C(3,-1,2)$ is .... *(1983 - 1 Mark)*

4. A, B, C and D, are four points in a plane with position vectors **a, b, c** and **d** respectively such that
$(\vec{a}-\vec{d})(\vec{b}-\vec{c})=(\vec{b}-\vec{d})(\vec{c}-\vec{a})=0$ *(1984 - 2 Marks)*
The point D, then, is the .... of the triangle ABC.

5. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and the vectors $\vec{A}=(1,a,a^2)$, $\vec{B}=(1,b,b^2)$, $\vec{C}=(1,c,c^2)$, are non-coplanar, then the product $abc=$ .... *(1985 - 2 Marks)*

6. If $\vec{A}\,\vec{B}\,\vec{C}$ are three non-coplanar vectors, then –
$\dfrac{\vec{A}\cdot\vec{B}\times\vec{C}}{\vec{C}\times\vec{A}\cdot\vec{B}}+\dfrac{\vec{B}\cdot\vec{A}\times\vec{C}}{\vec{C}\cdot\vec{A}\times\vec{B}}=$ .... *(1985 - 2 Marks)*

7. If $\vec{A}=(1,1,1)$, $\vec{C}=(0,1,-1)$ are given vectors, then a vector B satisfying the equations $\vec{A}\times\vec{B}=\vec{C}$ and $\vec{A}\cdot\vec{B}=3$ .... *(1985 - 2 Marks)*

8. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\dfrac{1}{(1-a)}+\dfrac{1}{(1-b)}+\dfrac{1}{(1-c)}=$ .... *(1987 - 2 Marks)*

9. Let $b = 4\hat{i}+3\hat{j}$ and $\vec{c}$ be two vectors perpendicular to each other in the xy-plane. All vectors in the same plane having projections 1 and 2 along $\vec{b}$ and $\vec{c}$, respectively, are given by .... *(1987 - 2 Marks)*

10. The components of a vector $\vec{a}$ along and perpendicular to a non-zero vector $\vec{b}$ are.......and ....respectively. *(1988 - 2 Marks)*

11. Given that $\vec{a}=(1,1,1)$, $\vec{c}=(0,1,-1)$, $\vec{a}.\vec{b}=3$ and $\vec{a}\times\vec{b}=\vec{c}$, then $\vec{b}=$ .... *(1991 - 2 Marks)*

12. A unit vector coplanar with $\vec{i}+\vec{j}+2\vec{k}$ and $\vec{i}+2\vec{j}+\vec{k}$ and perpendicular to $\vec{i}+\vec{j}+\vec{k}$ is ....*(1992 - 2 Marks)*

13. A unit vector perpendicular to the plane determined by the points $P(1,-1,2)$ $Q(2,0,-1)$ and $R(0,2,1)$ is .... *(1994 - 2 Marks)*

14. A nonzero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\vec{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is .... *(1996 - 2 Marks)*

15. If $\vec{b}$ and $\vec{c}$ are any two non-collinear unit vectors and $\vec{a}$ is any vector, then $(\vec{a}.\vec{b})\vec{b}+(\vec{a}.\vec{c})\vec{c}+\dfrac{\vec{a}.(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}(\vec{b}\times\vec{c})=$ .... *(1996 - 2 Marks)*

16. Let $OA=a$, $OB=10a+2b$ and $OC=b$ where O, A and C are non-collinear points. Let p denote the area of the quadrilateral $OABC$, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If $p=kq$, then $k=$ .... *(1997 - 2 Marks)*

B True / False

1. Let $\vec{A}, \vec{B}$ and $\vec{C}$ be unit vectors suppose that $\vec{A}.\vec{B}=\vec{A}.\vec{C}=0$, and that the angle between $\vec{B}$ and $\vec{C}$ is $\pi/6$. Then $\vec{A}=\pm 2(\vec{B}\times\vec{C})$. *(1981 - 2 Marks)*

2. If $X.A=0, X.B=0, X.C=0$ for some non-zero vector X, then $[A\,B\,C]=0$ *(1983 - 1 Mark)*
3. The points with position vectors $a+b, a-b$, and $a+kb$ are collinear for all real values of k. *(1984 - 1 Mark)*
4. For any three vectors $\vec{a}, \vec{b}$, and $\vec{c}$,

$(\vec{a}-\vec{b}).(\vec{b}-\vec{c})\times(\vec{c}-\vec{a}) = 2\vec{a}.\vec{b}\times\vec{c}$. *(1989 - 1 Mark)*

C MCQs with One Correct Answer

1. The scalar $\vec{A}.(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$ equals : *(1981 - 2 Marks)*
   (a) 0 (b) $[\vec{A}\ \vec{B}\ \vec{C}]+[\vec{B}\ \vec{C}\ \vec{A}]$
   (c) $[\vec{A}\ \vec{B}\ \vec{C}]$ (d) None of these
2. For non-zero vectors $\vec{a},\vec{b},\vec{c}$, $|(\vec{a}\times\vec{b}).\vec{c}|=|\vec{a}||\vec{b}||\vec{c}|$ holds if and only if *(1982 - 2 Marks)*
   (a) $\vec{a}.\vec{b}=0,\ \vec{b}.\vec{c}=0$ (b) $\vec{b}.\vec{c}=0,\ \vec{c}.\vec{a}=0$
   (c) $\vec{c}.\vec{a}=0\ \vec{a}.\vec{b}=0$ (d) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$
3. The volume of the parallelopiped whose sides are given by $\overrightarrow{OA}=2i-2j, \overrightarrow{OB}=i+j-k, \overrightarrow{OC}=3i-k$, is *(1983 - 1 Mark)*
   (a) $\frac{4}{13}$ (b) 4
   (c) $\frac{2}{7}$ (d) none of these
4. The points with position vectors $60i+3j, 40i-8j, ai-52j$ are collinear if *(1983 - 1 Mark)*
   (a) $a=-40$ (b) $a=40$
   (c) $a=20$ (d) none of these
5. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$, are vectors defined by the relations $\vec{p}=\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q}=\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r}=\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$ then the value of the expression $(\vec{a}+\vec{b}).\vec{p}+(\vec{b}+\vec{c}).\vec{q}+(\vec{c}+\vec{a}).\vec{r}$ is equal to *(1988 - 2 Marks)*
   (a) 0 (b) 1 (c) 2 (d) 3.
6. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is *(1993 - 1 Marks)*
   (a) the Arithmetic Mean of a and b
   (b) the Geometric Mean of a and b
   (c) the harmonic Mean of a and b
   (d) equal to zero
7. Let $\vec{p}$ and $\vec{q}$ be the position vectors of P and Q respectively, with respect to O and $|\vec{p}|=p, |\vec{q}|=q$. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendicular then *(1994)*
   (a) $9q^2=4q^2$ (b) $4p^2=9q^2$
   (c) $9p=4q$ (d) $4p=9q$
8. Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}, \beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}, \gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$ *(1994)*
   (a) are collinear
   (b) form an equilateral triangle
   (c) form a scalene triangle
   (d) form a right angled triangle
9. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{j}-\hat{k}, \vec{c}=\hat{k}-\hat{i}$. If $\vec{d}$ is a unit vector such that $\vec{a}.\vec{d}=0=[\vec{b}\ \vec{c}\ \vec{d}]$, then $\vec{d}$ equals *(1995S)*
   (a) $\pm\frac{\hat{i}+\hat{j}-2\hat{k}}{\sqrt{6}}$ (b) $\pm\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$
   (c) $\pm\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (d) $\pm\hat{k}$
10. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{(\vec{b}+\vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is *(1995S)*
    (a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\pi/2$ (d) π
11. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u}+\vec{v}+\vec{w}=0$. If $|\vec{u}|=3, |\vec{v}|=4$ and $|\vec{w}|=5$, then $\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}$ is *(1995S)*
    (a) 47 (b) -25 (c) 0 (d) 25
12. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non coplanar vectors, then *(1995S)* $(\vec{a}+\vec{b}+\vec{c}).[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$ equals
    (a) 0 (b) $[\vec{a}\ \vec{b}\ \vec{c}]$
    (c) $2[\vec{a}\ \vec{b}\ \vec{c}]$ (d) $-[\vec{a}\ \vec{b}\ \vec{c}]$
13. Let $\boldsymbol{a}=2\boldsymbol{i}+\boldsymbol{j}-2\boldsymbol{k}$ and $\boldsymbol{b}=\boldsymbol{i}+\boldsymbol{j}$. If c is a vector such that $\boldsymbol{a}.\boldsymbol{c}=|\boldsymbol{c}|, |\boldsymbol{c}-\boldsymbol{a}|=2\sqrt{2}$ and the angle between $(\boldsymbol{a}\times\boldsymbol{b})$ and $\boldsymbol{c}$ is 30°, then $|(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}|=$ *(1999 - 2 Marks)*
    (a) 2/3 (b) 3/2 (c) 2 (d) 3
14. Let $a=2i+j+k, b=i+2j-k$ and a unit vector c be coplanar. If c is perpendicular to a, then $c=$ *(1999 - 2 Marks)*
    (a) $\frac{1}{\sqrt{2}}(-j+k)$ (b) $\frac{1}{\sqrt{3}}(-i-j-k)$
    (c) $\frac{1}{\sqrt{5}}(i-2j)$ (d) $\frac{1}{\sqrt{3}}(i-j-k)$

15. If the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ form the sides BC, CA and AB respectively of a triangle ABC, then **(2000S)**

(a) $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$ (b) $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

(c) $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$ (d) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{0}$

16. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)=0$. Let P_1 and P_2 be planes determined by the pairs of vectors $\vec{a}, \vec{b}$ and $\vec{c}, \vec{d}$ respectively. Then the angle between P_1 and P_2 is **(2000S)**

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

17. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $\left[2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a}\right]=$ **(2000S)**

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

18. Let $\vec{a}=\vec{i}-\vec{k}$, $\vec{b}=x\vec{i}+\vec{j}+(1-x)\vec{k}$ and $\vec{c}=y\vec{i}+x\vec{j}+(1+x-y)\vec{k}$. Then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ depends on **(2001S)**

(a) only x (b) only y
(c) Neither x Nor y (d) both x and y

19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2$ does NOT exceed **(2001S)**

(a) 4 (b) 9 (c) 8 (d) 6

20. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+2\vec{b}$ and $5\vec{a}-4\vec{b}$ are perpendicular to each other then the angle between $\vec{a}$ and $\vec{b}$ is **(2002S)**

(a) 45° (b) 60°

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

21. Let $\vec{V}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{W}=\vec{i}+3\vec{k}$. If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $|\vec{U}\vec{V}\vec{W}|$ is

(a) –1 (b) $\sqrt{10}+\sqrt{6}$ **(2002S)**

(c) $\sqrt{59}$ (d) $\sqrt{60}$

22. The value of k such that $\frac{x-4}{1}=\frac{y-2}{1}=\frac{z-k}{2}$ lies in the plane $2x-4y+z=7$, is **(2003S)**

(a) 7 (b) –7
(c) no real value (d) 4

23. The value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is **(2003S)**

(a) –3 (b) 3 (c) $1/\sqrt{3}$ (d) $\sqrt{3}$

24. If $\vec{a}=(\hat{i}+\hat{j}+\hat{k}), \vec{a}.\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$, then $\vec{b}$ is

(a) $\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{j}-\hat{k}$ **(2004S)**

(c) $\hat{i}$ (d) $2\hat{i}$

25. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then the value of k is **(2004S)**

(a) 3/2 (b) 9/2 (c) –2/9 (d) –3/2

26. The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with the vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is **(2004S)**

(a) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (b) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

(c) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ (d) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

27. A variable plane at a distance of the one unit from the origin cuts the coordinates axes at A, B and C. If the centroid $D(x, y, z)$ of triangle ABC satisfies the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then the value k is **(2005S)**

(a) 3 (b) 1 (c) $\frac{1}{3}$ (d) 9

28. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, non-coplanar vectors and

$$\vec{b}_1=\vec{b}-\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a},\ \vec{b}_2=\vec{b}+\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a},$$

$$\vec{c}_1=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}+\frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1,\ \vec{c}_2=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{b}_1\cdot\vec{c}}{|\vec{b}_1|^2}\vec{b}_1,$$

$$\vec{c}_3=\vec{c}-\frac{\vec{c}\cdot\vec{a}}{|\vec{c}|^2}\vec{a}+\frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1,\ \vec{c}_4=\vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{c}|^2}\vec{a}=\frac{\vec{b}.\vec{c}}{|\vec{b}|^2}\vec{b}_1,$$

then the set of orthogonal vectors is **(2005S)**

(a) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (b) $(\vec{a}, \vec{b}_1, \vec{c}_2)$

(c) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (d) $(\vec{a}, \vec{b}_2, \vec{c}_2)$

29. A plane which is perpendicular to two planes $2x-2y+z=0$ and $x-y+2z=4$, passes through (1, –2, 1). The distance of the plane from the point (1, 2, 2) is **(2006 - 3M, –1)**

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

30. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is *(2006 - 3M, –1)*

(a) $4\hat{i} - \hat{j} + 4\hat{k}$ (b) $3\hat{i} + \hat{j} - 3\hat{k}$

(c) $2\hat{i} + \hat{j} - 2\hat{k}$ (d) $4\hat{i} + \hat{j} - 4\hat{k}$

31. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is *(2007 -3 marks)*

(a) zero (b) one (c) two (d) three

32. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which one of the following is correct ? *(2007 -3 marks)*

(a) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$

(b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$

(c) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} \neq \vec{0}$

(d) $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$ are mutually perpendicular

33. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = \frac{1}{2}$. Then, the volume of the parallelopiped is *(2008)*

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

34. Let two non-collinear unit vectors $\hat{a}$ and $\hat{b}$ form an acute angle. A point P moves so that at any time t the position vector $\overline{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overline{OP}$ and $\hat{u}$ be the unit vector along $\overline{OP}$. Then, *(2008)*

(a) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ *and* $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$

(b) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ *and* $M = (1 + \hat{a}\cdot\hat{b})^{1/2}$

(c) $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$ *and* $M = (1 + 2\hat{a}\cdot\hat{b})^{1/2}$

(d) $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$ *and* $M = (1 + 2\hat{a}\cdot\hat{b})^{1/2}$

35. Let $P(3, 2, 6)$ be a point in space and Q be a point on the line

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$$

Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is *(2009)*

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $-\frac{1}{8}$

36. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$, then *(2009)*

(a) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

(b) $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar

(c) $\vec{b}, \vec{d}$ are non-parallel

(d) $\vec{a}, \vec{d}$ are parallel and $\vec{b}, \vec{c}$ are parallel

37. A line with positive direction cosines passes through the point $P(2, -1, 2)$ and makes equal angles with the coordinate axes. The line meets the plane

$$2x + y + z = 9$$

at point Q. The length of the line segment PQ equals *(2009)*

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

38. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}, 4\hat{i}, 3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a *(2010)*

(a) parallelogram, which is neither a rhombus nor a rectangle

(b) square

(c) rectangle, but not a square

(d) rhombus, but not a square

39. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is *(2010)*

(a) $x + 2y - 2z = 0$ (b) $3x + 2y - 2z = 0$

(c) $x - 2y + z = 0$ (d) $5x + 2y - 4z = 0$

40. If the distance of the point $P(1, -2, 1)$ from the plane $x + 2y - 2z = \alpha$, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is *(2010)*

(a) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ (b) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$

(c) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ (d) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

41. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + 2\hat{k}$

The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by *(2010)*

(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

42. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three vectors. A vector $\vec{v}$ in the plane of $\vec{a}$ and $\vec{b}$, whose projection on $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by *(2011)*

(a) $\hat{i}-3\hat{j}+3\hat{k}$ (b) $-3\hat{i}-3\hat{j}-\hat{k}$
(c) $3\hat{i}-\hat{j}+3\hat{k}$ (d) $\hat{i}+3\hat{j}-3\hat{k}$

43. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is *(2012)*
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

44. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1 ,–1) is *(2012)*
(a) $5x - 11y + z = 17$ (b) $\sqrt{2}x+y=3\sqrt{2}-1$
(c) $x+y+z=\sqrt{3}$ (d) $x-\sqrt{2}y=1-\sqrt{2}$

45. If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}+\vec{b}\right|=\sqrt{29}$ and $\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\vec{b}$, then a possible value of $\left(\vec{a}+\vec{b}\right).\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is *(2012)*
(a) 0 (b) 3 (c) 4 (d) 8

46. Let P be the image of the point (3,1,7) with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through P and containing the straight line $\frac{x}{1}=\frac{y}{z}=\frac{z}{1}$ is *(JEE Adv. 2016)*
(a) $x + y - 3z = 0$ (b) $3x + z = 0$
(c) $x - 4y + 7z = 0$ (d) $2x - y = 0$

D MCQs with One or More than One Correct

1. Let $\vec{a}=a_1i+a_2j+a_3k$, $\vec{b}=b_1i+b_2j+b_3k$ and $\vec{c}=c_1i+c_2j+c_3k$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to *(1986 - 2 Marks)*
(a) 0
(b) 1
(c) $\frac{1}{4}(a_1^2+a_2^2+a_2^3)\ (b_1^2+b_2^2+b_3^2)$
(d) $\frac{3}{4}(a_1^2+a_2^2+a_3^2)\ (b_1^2+b_2^2+b_3^2)\ (c_1^2+c_2^2+c_3^2)$

2. The number of vectors of unit length perpendicular to vectors $\vec{a}=(1, 1, 0)$ and $\vec{b}=(0, 1, 1)$ is *(1987 - 2 Marks)*
(a) one (b) two (c) three (d) infinite
(e) None of these.

3. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}-2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3},$ is : *(1993 - 2 Marks)*
(a) $2\hat{i}+3\hat{j}-3\hat{k}$ (b) $2\hat{i}+3\hat{j}+3\hat{k}$
(c) $-2\hat{i}-\hat{j}+5\hat{k}$ (d) $2\hat{i}+\hat{j}+5\hat{k}$

4. The vector $\frac{1}{3}(2\hat{i}-2\hat{j}+\hat{k})$ is *(1994)*
(a) a unit vecotr
(b) makes an angle $\frac{\pi}{3}$ with the vector $(2\hat{i}-4\hat{j}+3\hat{k})$
(c) parallel to the vector $\left(-\hat{i}+\hat{j}-\frac{1}{2}\hat{k}\right)$
(d) perpendicular to the vector $3\hat{i}+2\hat{j}-2\hat{k}$

5. If $a=i+j+k, \vec{b}=4i+3j+4k$ and $c=i+\alpha j+\beta k$ are linearly dependent vectors and $|c|=\sqrt{3}$, then *(1998 - 2 Marks)*
(a) $\alpha=1, \beta=-1$ (b) $\alpha=1, \beta=\pm1$
(c) $\alpha=-1, \beta=\pm1$ (d) $\alpha=\pm1, \beta=1$

6. For three vectors u, v, w which of the following expression is not equal to any of the remaining three? *(1998 - 2 Marks)*
(a) $u\bullet(v\times w)$ (b) $(v\times w)\bullet u$
(c) $v\bullet(u\times w)$ (d) $(u\times v)\bullet w$

7. Which of the following expressions are meaningful? *(1998 - 2 Marks)*
(a) $u\,(v\times w)$ (b) $(u\bullet v)\bullet w$
(c) $(u\bullet v)\,w$ (d) $u\times(v\bullet w)$

8. Let a and b be two non-collinear unit vectors. If $u = a - (a\,.\,b)\,b$ and $v = a\times b$, then $|v|$ is *(1999 - 3 Marks)*
(a) $|u|$ (b) $|u|+|u\,.\,a|$
(c) $|u|+|u\,.\,b|$ (d) $|u|+u.(a+b)$

9. Let $\vec{A}$ be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and that P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between vector $\vec{A}$ and a given vector $2\hat{i}+\hat{j}-2\hat{k}$ is *(2006 - 5M, –1)*
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

10. The vector (s) which is/are coplanar with vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to the vector $\hat{i}+\hat{j}+\hat{k}$ is/are *(2011)*
(a) $\hat{j}-\hat{k}$ (b) $-\hat{i}+\hat{j}$ (c) $\hat{i}-\hat{j}$ (d) $-\hat{j}+\hat{k}$

11. If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane (s) containing these two lines is (are) *(2012)*

(a) $y+2z=-1$ (b) $y+z=-1$
(c) $y-z=-1$ (d) $y-2z=-1$

12. A line l passing through the origin is perpendicular to the lines *(JEE Adv. 2013)*

$$l_1:(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}, \quad -\infty<t<\infty$$

$$l_2:(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}, \quad -\infty<s<\infty$$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

(a) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (b) $(-1,-1,0)$
(c) $(1,1,1)$ (d) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

13. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) *(JEE Adv. 2013)*

(a) 1 (b) 2 (c) 3 (d) 4

14. Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\vec{a}$ is a non-zero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a non-zero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then *(JEE Adv. 2014)*

(a) $\vec{b}=\left(\vec{b}\cdot\vec{z}\right)\left(\vec{z}-\vec{x}\right)$

(b) $\vec{a}=\left(\vec{a}\cdot\vec{y}\right)\left(\vec{y}-\vec{z}\right)$

(c) $\vec{a}\cdot\vec{b}=-\left(\vec{a}\cdot\vec{y}\right)\left(\vec{b}\cdot\vec{z}\right)$

(d) $\vec{a}=-\left(\vec{a}\cdot\vec{y}\right)\left(\vec{z}-\vec{y}\right)$

15. From a point $P(\lambda,\lambda,\lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y=x, z=1$ and $y=-x, z=-1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) *(JEE Adv. 2014)*

(a) $\sqrt{2}$ (b) 1 (c) -1 (d) $-\sqrt{2}$

16. In R^3, consider the planes $P_1: y=0$ and $P_2: x+z=1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relations is (are) true? *(JEE Adv. 2015)*

(a) $2\alpha+\beta+2\gamma+2=0$ (b) $2\alpha-\beta+2\gamma+4=0$
(c) $2\alpha+\beta-2\gamma-10=0$ (d) $2\alpha-\beta+2\gamma-8=0$

17. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z+1=0$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie (s) on M? *(JEE Adv. 2015)*

(a) $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ (b) $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$
(c) $\left(-\frac{5}{6},0,\frac{1}{6}\right)$ (d) $\left(-\frac{1}{3},0,\frac{2}{3}\right)$

18. Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$, $\vec{b}\cdot\vec{c}=24$, then which of the following is (are) true? *(JEE Adv. 2015)*

(a) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$ (b) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$
(c) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$ (d) $\vec{a}\cdot\vec{b}=-72$

19. Consider a pyramid OPQRS located in the first octant ($x\geq 0, y\geq 0, z\geq 0$) with O as origin, and OP and OR along the x–axis and the y–axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point, T of diagonal OQ such that TS = 3. Then *(JEE Adv. 2016)*

(a) the acute angle between OQ and OS is $\frac{\pi}{3}$
(b) the equation of the plane containing the triangle OQS is $x-y=0$
(c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
(d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

20. Let $\hat{u}=u_1\hat{i}+u_2\hat{j}+u_3\hat{k}$ be a unit vector in R^3 and $\hat{w}=\frac{1}{\sqrt{6}}\left(\hat{i}+\hat{j}+2\hat{k}\right)$. Given that there exists a vector $\vec{v}$ in R^3 such that $\left|\hat{u}\times\vec{v}\right|=1$ and $\hat{w}\cdot\left(\hat{u}\times\vec{v}\right)=1$. Which of the following statement(s) is (are) correct? *(JEE Adv. 2016)*

(a) There is exactly one choice for such $\vec{v}$

(b) There are infinitely many choices for such $\vec{v}$

(c) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$

(d) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

E Subjective Problems

1. From a point O inside a triangle ABC, perpendiculars OD, OE, OF are drawn to the sides BC, CA, AB respectively. Prove that the perpendiculars from A, B, C to the sides EF, FD, DE are concurrent. *(1978)*

2. $A_1, A_2, \ldots\ldots A_n$ are the vertices of a regular plane polygon with n sides and O is its centre. Show that

$$\sum_{i=1}^{n-1}(\overrightarrow{OA_i} \times \overrightarrow{OA}_{i+1}) = (1-n)(\overrightarrow{OA_2} \times \overrightarrow{OA_1})$$ *(1982 - 2 Marks)*

3. Find all values of λ such that $x, y, z, \neq (0,0,0)$ and

$(\vec{i} + \vec{j} + 3\vec{k})x + (3\vec{i} - 3\vec{j} + \vec{k})y + (-4\vec{i} + 5\vec{j})z$

$= \lambda(x\vec{i} \times \vec{j}\, y + \vec{k}\, z)$ where $\vec{i}$, $\vec{j}$, $\vec{k}$ are unit vectors along the coordinate axes. *(1982 - 3 Marks)*

4. A vector $\vec{A}$ has components A_1, A_2, A_3 in a right -handed rectangular Cartesian coordinate system $oxyz$. The coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$. Find the components of A in the new coordinate system, in terms of A_1, A_2, A_3. *(1983 - 2 Marks)*

5. The position vectors of the points A, B, C and D are $3\hat{i} - 2\hat{j} - \hat{k}, 2\hat{i} + 3\hat{j} - 4\hat{k}, -\hat{i} + \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$, respectively. If the points A, B, C and D lie on a plane, find the value of λ. *(1986 - 2½ Marks)*

6. If A, B, C, D are any four points in space, prove that – *(1987 - 2 Marks)*

$\left|\overrightarrow{AB} \times \overrightarrow{CD} + \overrightarrow{BC} \times \overrightarrow{AD} + \overrightarrow{CA} \times \overrightarrow{BD}\right| = 4$ (area of triangle ABC)

7. Let $OA\,CB$ be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. Using vector methods prove that BD and CO intersect in the same ratio. Determine this ratio. *(1988 - 3 Marks)*

8. If vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = \vec{0}$$ *(1989 - 2 Marks)*

9. In a triangle OAB, E is the midpoint of BO and D is a point on AB such that $AD : DB = 2 : 1$. If OD and AE intersect at P, determine the ratio $OP : PD$ using vector methods. *(1989 - 4 Marks)*

10. Let $\vec{A} = 2\vec{i} + \vec{k}$, $\vec{B} = \vec{i} + \vec{j} + \vec{k}$, and $\vec{C} = 4\vec{i} - 3\vec{j} + 7\vec{k}$. Determine a vector $\vec{R}$. Satisfying $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = 0$ *(1990 - 3 Marks)*

11. Determine the value of 'c' so that for all real x, the vector $cx\hat{i} - 6\hat{j} - 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other. *(1991 - 4 Marks)*

12. In a triangle ABC, D and E are points on BC and AC respectively, such that $BD = 2\,DC$ and $AE = 3EC$. Let P be the point of intersection of AD and BE. Find BP/PE using vector methods. *(1993 - 5 Marks)*

13. If the vectors $\vec{b}, \vec{c}, \vec{d}$, are not coplanar, then prove that the vector $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$ is parallel to $\vec{a}$. *(1994 - 4 Marks)*

14. The position vectors of the vertices A, B and C of a tetrahedron $ABCD$ are $\hat{i} + \hat{j} + \hat{k}$, $\hat{i}$ and $3\hat{i}$, respectively. The altitude from vertex D to the opposite face ABC meets the median line through A of the triangle ABC at a point E. If the length of the side AD is 4 and the volume of the tetrahedron is $\frac{2\sqrt{2}}{3}$, find the position vector of the point E for all its possible positions. *(1996 - 5 Marks)*

15. If A, B and C are vectors such that $|B| = |C|$. Prove that $[(A+B) \times (A+C)] \times (B \times C)(B+C) = 0$. *(1997 - 5 Marks)*

16. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid-points of the parallel sides. (You may assume that the trapezium is not a parallelogram.) *(1998 - 8 Marks)*

17. For any two vectors u and v, prove that *(1998 - 8 Marks)*
(a) $(u \cdot v)^2 + |u \times v|^2 = |u|^2 |v|^2$ and
(b) $(1+|u|^2)(1+|v|^2) = (1 - u \cdot v)^2 + |u + v + (u \times v)|^2$.

18. Let u and v be unit vectors. If w is a vector such that $w + (w \times u) = v$, then prove that $|(u \times v) \cdot w| \leq 1/2$ and that the equality holds if and only if u is perpendicular to v. *(1999 - 10 Marks)*

19. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices. *(2001 - 5 Marks)*

20. Find 3–dimensional vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ satisfying
$\vec{v}_1.\vec{v}_1 = 4, \vec{v}_1.\vec{v}_2 = -2, \vec{v}_1.\vec{v}_3 = 6, \vec{v}_2.\vec{v}_2 = 2, \vec{v}_2.\vec{v}_3 = -5, \vec{v}_3.\vec{v}_3 = 29$ *(2001 - 5 Marks)*

21. Let $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$ and

$\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}, t \in [0,1]$,

where f_1, f_2, g_1, g_2 are continuous functions. If $\vec{A}(t)$ and $\vec{B}(t)$ are nonzero vectors for all t and $\vec{A}(0) = 2\hat{i} + 3\hat{j}$, $\vec{A}(1) = 6\hat{i} + 2\hat{j}$, $\vec{B}(0) = 3\hat{i} + 2\hat{j}$ and $\vec{B}(1) = 2\hat{i} + 6\hat{j}$. Then show that $\vec{A}(t)$ and $\vec{B}(t)$ are parallel for some t. *(2001 - 5 Marks)*

22. Let V be the volume of the parallelopiped formed by the vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. If a_r, b_r, c_r, where $r = 1, 2, 3$, are non-negative real numbers and $\sum_{r=1}^{3}(a_r + b_r + c_r) = 3L$, show that $V \le L^3$. *(2002 - 5 Marks)*

23. (i) Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. *(2003 - 4 Marks)*

24. If $\vec{u}, \vec{v}, \vec{w}$, are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that

$$[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}] = \frac{1}{16}[\vec{u}\ \vec{v}\ \vec{w}]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}.$$

(2003 - 4 Marks)

25. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are distinct vectors such that $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$. Prove that $(\vec{a}-\vec{d}).(\vec{b}-\vec{c}) \ne 0$ i.e. $\vec{a}.\vec{b} + \vec{d}.\vec{c} \ne \vec{d}.\vec{b} + \vec{a}.\vec{c}$ *(2004 - 2 Marks)*

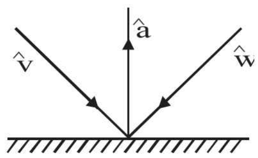
26. Find the equation of plane passing through (1, 1, 1) & parallel to the lines L_1, L_2 having direction ratios (1,0,–1), (1,–1,0). Find the volume of tetrahedron formed by origin and the points where these planes intersect the coordinate axes. *(2004 - 2 Marks)*

27. A parallelopiped 'S' has base points A, B, C and D and upper face points A', B', C' and D'. This parallelopiped is compressed by upper face $A'B'C'D'$ to form a new parallelopiped 'T' having upper face points A'', B'', C'' and D''. Volume of parallelopiped T is 90 percent of the volume of parallelopiped S. Prove that the locus of 'A''', is a plane. *(2004 - 2 Marks)*

28. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two line on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C, whose permutation A', B', C' can be chosen such that (i) A is on L_1, B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2 *(2004 - 4 Marks)*

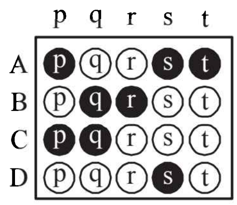
29. Find the equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1). *(2005 - 2 Marks)*

30. If the incident ray on a surface is along the unit vector $\hat{v}$, the reflected ray is along the unit vector $\hat{w}$ and the normal is along unit vector $\hat{a}$ outwards. Express $\hat{w}$ in terms of $\hat{a}$ and $\hat{v}$. *(2005 - 4 Marks)*



F Match the Following

DIRECTIONS (Q. 1-6) : *Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :*
If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



1. Match the following : *(2006 - 6M)*

(A) Two rays $x + y = |a|$ and $ax - y = 1$ intersects each other in the first quadrant in the interval $a \in (a_0, \infty)$, the value of a_0 is — (p) 2

(B) Point (α, β, γ) lies on the plane $x + y + z = 2$. Let $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\hat{k}\times(\hat{k}\times\vec{a}) = 0$, then $\gamma =$ — (q) $\frac{4}{3}$

(C) $\left|\int_0^1 (1 - y^2)\,dy\right| + \left|\int_1^0 (y^2 - 1)\,dy\right|$ — (r) $\left|\int_0^1 \sqrt{1-x}\,dx\right| + \left|\int_{-1}^0 \sqrt{1+x}\,dx\right|$

(D) If $\sin A \sin B \sin C + \cos A \cos B = 1$, then the value of $\sin C =$ — (s) 1

2. Consider the following linear equations

$ax + by + cz = 0; \quad bx + cy + az = 0; \quad cx + ay + bz = 0$

Match the conditions/expressions in **Column I** with statements in **Column II** and indicate your answer by darkening the appropriate bubbles in the 4 × 4 matrix given in the *ORS*. *(2007)*

Column I	Column II
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at asingle point
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q) the equations represent the line $x = y = z$.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes.
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s) the equations represent the whole of the three dimensional space.

3. Match the statements / expressions given in **Column-I** with the values given in **Column-II**. *(2009)*

Column-I	Column-II
(A) Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p) $\frac{\pi}{6}$
(B) Points of discontinuity of the unction $f(x) = \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$, f where $[y]$ denotes the largest integer less than or equal to y	(q) $\frac{\pi}{4}$
(C) Volume of the parallelopiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$	(r) $\frac{\pi}{3}$
(D) Angle between vector $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \sqrt{3}\,\vec{c} = \vec{0}$	(s) $\frac{\pi}{2}$
	(t) π

4. Match the statements/expressions given in **Column-I** with the values given in **Column-II**. *(2009)*

Column-I	Column-II
(A) The number of solutions of the equation $x\,e^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p) 1
(B) Value(s) of k for which the planes $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	(q) 2
(C) Value(s) of k for which $\|x-1\| + \|x-2\| + \|x+1\| \| + \|x+2\| = 4k$ has integer solution(s)	(r) 3
(D) If $y' = y + 1$ and $y(0) = 1$, then value(s) of y (1n 2)	(s) 4
	(t) 5

5. Match the statement in **Column-1** with the values in **Column -II** *(2010)*

Column – I	Column – II
(A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x - \frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If length $PQ = d$, then d^2 is	(p) –4
(B) The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\left(\frac{3}{5}\right)$ are	(q) 0
(C) Non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$. $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$ and $2\|\vec{b} + \vec{c}\| = \|\vec{b} - \vec{a}\|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the possible values of μ are	(r) 4

(D) Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$

and $f(x) = \sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ *for* $x \neq 0$ — (s) 5

The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is

(t) 6

6. Match the statements given in Column-I with the values given in Column-II. *(2011)*

Column-I	Column-II
(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x)\, dx = a^2 - b^2$, then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\ell n 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is	(r) $\frac{\pi}{3}$
(D) The maximum value of $\left|Arg\left(\frac{1}{1-z}\right)\right|$ for $\lvert z\rvert = 1, z \neq 1$ is given by	(s) π
	(t) $\frac{\pi}{2}$

DIRECTIONS (Q. 7-9) : *Each question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.*

7. Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2013)*

List I	List II
P. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $2(\vec{c} \times \vec{a})$ is	1. 100
Q. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	2. 30
R. Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$ is	3. 24
S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	4. 60

Codes:

	P	Q	R	S
(a)	4	2	3	1
(b)	2	3	1	4
(c)	3	4	1	2
(d)	1	4	3	2

8. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}, L_2 \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3, P_2 = 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2013)*

	List I		List II
P.	$a =$	1.	13
Q.	$b =$	2.	-3
R.	$c =$	3.	1
S.	$d =$	4.	-2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

9. Match List I with List II and select the correct answer using the code given below the lists : *(JEE Adv. 2014)*

List - I / **List - II**

P. Let $y(x) = \cos(3\cos^{-1} x)$, $x \in [-1, 1]$, $x \neq \pm\frac{\sqrt{3}}{2}$. Then

$\frac{1}{y(x)}\left\{(x^2 - 1)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx}\right\}$ equals — 1. 1

Q. Let $A_1, A_2, ..., A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k = 1, 2, ..., n$.

If $\left|\sum_{k=1}^{n-1}\left(\vec{a}_k \times \vec{a}_{k+1}\right)\right| = \left|\sum_{k=1}^{n-1}\left(\vec{a}_k \cdot \vec{a}_{k+1}\right)\right|$,

then the minimum value of n is — 2. 2

R. If the normal from the point $P(h, 1)$ on the ellipse $\frac{x^2}{6} + \frac{y^2}{3} = 1$ is perpendicular to the line $x + y = 8$, then the value of h is — 3. 8

S. Number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$ is — 4. 9

	P	Q	R	S
(a)	4	3	2	1
(b)	2	4	3	1
(c)	4	3	1	2
(d)	2	4	1	3

DIRECTIONS (Q. 10 & 11) : *Refer to Directions (1-6).*

10. Match the following : *(JEE Adv. 2015)*

Column I	Column II
(A) In R^2, if the magnitude of the projection vector of the vector $\alpha\hat{i}+\beta\hat{j}$ on $\sqrt{3}\hat{i}+\hat{j}$ is $\sqrt{3}$ and if $\alpha = 2+\sqrt{3}\beta$, then possible value of $\|\alpha\|$ is/are	(p) 1
(B) Let a and b be real numbers such that the function $f(x)=\begin{cases}-3ax^2-2, & x<1\\ bx+a^2, & x\ge 1\end{cases}$ if differentiable for all $x \in R$. Then possible value of a is (are)	(q) 2
(C) Let $\omega \neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+3\omega^2)^{4n+3}=0$, then possible value (s) of n is (are)	(r) 3
(D) Let the harmonic mean of two positive real numbers a and b be 4. If q is a positive real number such that a, 5, q, b is an arithmetic progression, then the value(s) of $\|q-a\|$ is (are)	(s) 4
	(t) 5

11. Match the following : *(JEE Adv. 2015)*

Column I	Column II
(A) In a triangle ΔXYZ, let a, b, and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2(a^2-b^2)=c^2$ and $\lambda=\dfrac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\pi\lambda)=0$ is (are)	(p) 1
(B) In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y, and Z respectively. If $1+\cos 2X-2\cos 2Y = 2\sin X\sin Y$, then possible value (s) of $\dfrac{a}{b}$ is (are)	(q) 2
(C) In R^2, let $\sqrt{3}i+\hat{j}, \hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect to the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\dfrac{3}{\sqrt{2}}$, then possible value(s) of $\|\beta\|$ is (are)	(r) 3
(D) Suppose that $F(\alpha)$ denotes the area of the region bounded by $x=0$, $x=2$, $y^2=4x$ and $y=\|\alpha x-1\|+\|\alpha x-2\|+\alpha x$, where $\alpha \in \{0, 1\}$. Then the value(s) of $F(\alpha)+\dfrac{8}{3}\sqrt{2}$, when $\alpha=0$ and $\alpha=1$, is (are)	(s) 5
	(t) 6

G Comprehension Based Questions

Consider the lines

$$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2} \quad L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$$

1. The unit vector perpendicular to both L_1 and L_2 is *(2008)*
   (a) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (b) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$
   (c) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (d) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$
2. The shortest distance between L_1 and L_2 is *(2008)*
   (a) 0 (b) $\frac{17}{\sqrt{3}}$
   (c) $\frac{41}{5\sqrt{3}}$ (d) $\frac{17}{5\sqrt{3}}$
3. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is *(2008)*
   (a) $\frac{2}{\sqrt{75}}$ (b) $\frac{7}{\sqrt{75}}$
   (c) $\frac{13}{\sqrt{75}}$ (d) $\frac{23}{\sqrt{75}}$

H Assertion & Reason Type Questions

1. Consider the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
   STATEMENT-1 : The parametric equations of the line of intersection of the given planes are $x = 3 + 14t, y = 1 + 2t, z = 15t$. **because**
   STATEMENT-2 : The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of given planes. *(2007 -3 marks)*
   (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
   (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (c) Statement-1 is True, Statement-2 is False
   (d) Statement-1 is False, Statement-2 is True.
2. Let the vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$ and $\overrightarrow{UP}$ represent the sides of a regular hexagon.
   STATEMENT-1 : $\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \vec{0}$. **because**
   STATEMENT-2 : $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq \vec{0}$. *(2007 -3 marks)*
   (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (c) Statement-1 is True, Statement-2 is False
   (d) Statement-1 is False, Statement-2 is True.
3. Consider three planes
   $P_1 : x - y + z = 1$ $\quad P_2 : x + y - z = 1$
   $P_3 : x - 3y + 3z = 2$
   Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, P_1 and P_2, respectively.
   STATEMENT - 1Z : At least two of the lines L_1, L_2 and L_3 are non-parallel and
   STATEMENT - 2 : The three planes doe not have a common point. *(2008)*
   (A) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
   (B) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 **is NOT** a correct explanation for STATEMENT - 1
   (C) STATEMENT - 1 is True, STATEMENT - 2 is False
   (D) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i}-2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$, then find the value of $\left(2\vec{a}+\vec{b}\right)$. $\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}-2\vec{b}\right)\right]$. *(2010)*
2. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then find $|d|$. *(2010)*
3. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is *(2011)*
4. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying *(2012)*
   $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is
5. Consider the set of eight vectors $V = \{a\hat{i}+b\hat{j}+c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is *(JEE Adv. 2013)*

6. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20= *(JEE Adv. 2013)*

7. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is *(JEE Adv. 2014)*

8. Suppose that $\vec{p},\vec{q}$ and $\vec{r}$ are three non-coplanar vectors in $\mathbb{R}^3$. Let the components of a vector $\vec{s}$ along $\vec{p},\vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r}),(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of $2x+y+z$ is

(JEE Adv. 2015)

Section-B JEE Main / AIEEE

1. A plane which passes through the point (3, 2, 0) and the line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$ is **[2002]**
   (a) $x-y+z=1$ (b) $x+y+z=5$
   (c) $x+2y-z=1$ (d) $2x-y+z=5$

2. If $|\vec{a}|=4, |\vec{b}|=2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi/6$ then $(\vec{a}\times\vec{b})^2$ is equal to **[2002]**
   (a) 48 (b) 16
   (c) $\vec{a}$ (d) none of these

3. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $[\vec{a}\ \vec{b}\ \vec{c}]=4$ then $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]=$ **[2002]**
   (a) 16 (b) 64 (c) 4 (d) 8

4. If $\vec{a}, \vec{b}, \vec{c}$ are vectors show that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is **[2002]**
   (a) 60° (b) 30° (c) 45° (d) 90°

5. If $|\vec{a}|=5$, $|\vec{b}|=4$, $|\vec{c}|=3$ thus what will be the value of $|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}|$, given that $\vec{a}+\vec{b}+\vec{c}=0$ **[2002]**
   (a) 25 (b) 50 (c) –25 (d) –50

6. If the vectors $\vec{c}$, $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\hat{b}=\hat{j}$ are such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is : **[2002]**
   (a) $z\hat{i}-x\hat{k}$ (b) $\vec{0}$
   (c) $y\hat{j}$ (d) $-z\hat{i}+x\hat{k}$

7. $\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $|\vec{a}|:|\vec{b}|:|\vec{c}|$
   (a) $\sqrt{34}:\sqrt{45}:\sqrt{39}$ (b) $\sqrt{34}:\sqrt{45}:39$ **[2002]**
   (c) 34 : 39 : 45 (d) 39 : 35 : 34

8. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$ **[2002]**
   (a) abc (b) –1 (c) 0 (d) 2

9. The d.r. of normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle $\pi/4$ with plane $x+y=3$ are **[2002]**
   (a) $1, \sqrt{2}, 1$ (b) $1, 1, \sqrt{2}$
   (c) 1, 1, 2 (d) $\sqrt{2}, 1, 1$

10. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ *and* $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}.\hat{n}=0$ and $\vec{v}.\hat{n}=0$, then $|\vec{w}.\hat{n}|$ is equal to **[2003]**
    (a) 3 (b) 0 (c) 1 (d) 2.

11. A particle acted on by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $\hat{i}+2\hat{j}-3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The total work done by the forces is
    (a) 50 units (b) 20 units **[2003]**
    (c) 30 units (d) 40 units.

12. The vectors $\overrightarrow{AB}=3\hat{i}+4\hat{k}$ & $\overrightarrow{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is **[2003]**
    (a) $\sqrt{288}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

13. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is

(a) 39 (b) 26 (c) $11\frac{4}{13}$ (d) 13

14. The two lines $x=ay+b, z=cy+d$ and $x=a`y+b`, z=c`y+d`$ will be perpendicular, if and only if **[2003]**

(a) $aa`+cc`+1=0$
(b) $aa`+bb`+cc`+1=0$
(c) $aa`+bb`+cc`=0$
(d) $(a+a`)(b+b`)+(c+c`)=0.$

15. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{1}=\frac{z-5}{1}$ are coplanar if **[2003]**

(a) k = 3 or –2 (b) k = 0 or –1
(c) k = 1 or –1 (d) k = 0 or –3

16. $\vec{a},\vec{b},\vec{c}$ are 3 vectors, such that $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=1$ $|\vec{b}|=2, |\vec{c}|=3$, , then $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is equal to **[2003]**

(a) 1 (b) 0 (c) –7 (d) 7

17. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is **[2003]**

(a) 4 (b) 1 (c) 2 (d) 3

18. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1) B(2, 1, 3) and C(–1, 1, 2). Then the angle between the faces OAB and ABC will be **[2003]**

(a) 90^o (b) $\cos^{-1}\left(\frac{19}{35}\right)$

(c) $\cos^{-1}\left(\frac{17}{31}\right)$ (d) 30^o

19. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$ and vectors $(1,a,a^2)$, $(1,b,b^2)$ and $(1,c,c^2)$ are non- coplanar, then the product abc equals **[2003]**

(a) 0 (b) 2 (c) –1 (d) 1

20. Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a **[2003]**

(a) parallelogram but not a rhombus
(b) square
(c) rhombus
(d) rectangle.

21. If $\vec{u},\vec{v}$ *and* $\vec{w}$ are three non- coplanar vectors, then $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}-\vec{v})\times(\vec{v}-\vec{w})$ equals **[2003]**

(a) $3\vec{u}.\vec{v}\times\vec{w}$ (b) 0
(c) $\vec{u}.\vec{v}\times\vec{w}$ (d) $\vec{u}.\vec{w}\times\vec{v}$

22. Two system of rectangular axes have the same origin. If a plane cuts them at distances a,b,c and a',b',c' from the origin then **[2003]**

(a) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(b) $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}=0$

(c) $\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}+\frac{1}{b'^2}-\frac{1}{c'^2}=0$

(d) $\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}+\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0.$

23. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y +4z + 5 = 0 is **[2004]**

(a) $\frac{9}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{3}{2}$

24. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x=y+a=z$ and $x+a=2y=2z$. The co-ordinates of each of the points of intersection are given by **[2004]**

(a) $(2a,3a,3a),(2a,a,a)$ (b) $(3a,2a,3a),(a,a,a)$
(c) $(3a,2a,3a),(a,a,2a)$ (d) $(3a,3a,3a),(a,a,a)$

25. If the straight lines **[2004]**

$x=1+s, y=-3-\lambda s, z=1+\lambda s$ and $x=\frac{t}{2}, y=1+t, z=2-t,$

with parameters s and t respectively, are co-planar, then λ equals.

(a) 0 (b) –1
(c) $-\frac{1}{2}$ (d) –2

26. The intersection of the spheres

$x^2+y^2+z^2+7x-2y-z=13$ and

$x^2+y^2+z^2-3x+3y+4z=8$

is the same as the intersection of one of the sphere and the plane **[2004]**

(a) $2x-y-z=1$ (b) $x-2y-z=1$
(c) $x-y-2z=1$ (d) $x-y-z=1$

27. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a}+2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b}+3\vec{c}$ is collinear with $\vec{a}$ (λ being some non-zero scalar) then $\vec{a}+2\vec{b}+6\vec{c}$ equals **[2004]**

(a) 0 (b) $\lambda\vec{b}$ (c) $\lambda\vec{c}$ (d) $\lambda\vec{a}$

28. A particles is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by **[2004]**

(a) 15 (b) 30 (c) 25 (d) 40

29. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a}+2\vec{b}+3\vec{c}$, $\lambda\vec{b}+4\vec{c}$ and $(2\lambda-1)\vec{c}$ are non coplanar for **[2004]**

(a) no value of λ
(b) all except one value of λ
(c) all except two values of λ
(d) all values of λ

30. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u}-\vec{v}+\vec{w}|$ equals **[2004]**

(a) 14 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 2

31. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals **[2004]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

32. If C is the mid point of AB and P is any point outside AB, then **[2005]**

(a) $\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$ (b) $\overrightarrow{PA}+\overrightarrow{PB}=\overrightarrow{PC}$

(c) $\overrightarrow{PA}+\overrightarrow{PB}+2\overrightarrow{PC}=\vec{0}$ (d) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{0}$

33. If the angel θ between the line $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}\,z+4=0$ is such that $\sin\theta=\frac{1}{3}$ then the value of λ is **[2005]**

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

34. The angle between the lines $2x=3y=-z$ and $6x=-y=-4z$ is **[2005]**

(a) 0° (b) 90°
(c) 45° (d) 30°

35. If the plane $2ax-3ay+4az+6=0$ passes through the midpoint of the line joining the centres of the spheres $x^2+y^2+z^2+6x-8y-2z=13$ and $x^2+y^2+z^2-10x+4y-2z=8$ then a equals **[2005]**

(a) -1 (b) 1
(c) -2 (d) 2

36. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(i-j+4k)$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ is **[2005]**

(a) $\frac{10}{9}$ (b) $\frac{10}{3\sqrt{3}}$

(c) $\frac{3}{10}$ (d) $\frac{10}{3}$

37. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to **[2005]**

(a) $3\vec{a}^2$ (b) $\vec{a}^2$

(c) $2\vec{a}^2$ (d) $4\vec{a}^2$

38. If non zero numbers a, b, c are in H.P., then the straight line $\frac{x}{a}+\frac{y}{b}+\frac{1}{c}=0$ always passes through a fixed point. That point is **[2005]**

(a) $(-1,2)$ (b) $(-1,-2)$

(c) $(1,-2)$ (d) $\left(1,-\frac{1}{2}\right)$

39. Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then c is **[2005]**

(a) the Geometric Mean of a and b
(b) the Arithmetic Mean of a and b
(c) equal to zero
(d) the Harmonic Mean of a and b

40. If $\vec{a},\vec{b},\vec{c}$ are non coplanar vectors and λ is a real number then $[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \lambda\vec{c}]=[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$ for **[2005]**

(a) exactly one value of λ
(b) no value of λ
(c) exactly three values of λ
(d) exactly two values of λ

41. Let $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$. Then $[\vec{a},\vec{b},\vec{c}]$ depends on **[2005]**

(a) only y (b) only x
(c) both x and y (d) neither x nor y

42. The plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius **[2005]**

(a) 3 (b) 1 (c) 2 (d) $\sqrt{2}$

43. If $(\bar{a}\times\bar{b})\times\bar{c}=\bar{a}\times(\bar{b}\times\bar{c})$ where $\bar{a},\bar{b}$ and $\bar{c}$ are any three vectors such that $\bar{a}.\bar{b}\neq 0$, $\bar{b}.\bar{c}\neq 0$ then $\bar{a}$ and $\bar{c}$ are **[2006]**

(a) inclined at an angle of $\frac{\pi}{3}$ between them
(b) inclined at an angle of $\frac{\pi}{6}$ between them
(c) perpendicular
(d) parallel

44. The values of a, for which points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $a\hat{i}-3\hat{j}+\hat{k}$ respectively are the vertices of a right angled triangle with $C=\frac{\pi}{2}$ are **[2006]**

(a) 2 and 1 (b) – 2 and – 1
(c) – 2 and 1 (d) 2 and – 1

45. The two lines $x=ay+b$, $z=cy+d$; and $x=a'y+b'$, $z=c'y+d'$ are perpendicular to each other if **[2006]**

(a) $aa'+cc'=-1$ (b) $aa'+cc'=1$
(c) $\frac{a}{a'}+\frac{c}{c'}=-1$ (d) $\frac{a}{a'}+\frac{c}{c'}=1$

46. The image of the point (–1, 3, 4) in the plane $x-2y=0$ is **[2006[**

(a) $\left(-\frac{17}{3},-\frac{19}{3},4\right)$ (b) $(15,11,4)$
(c) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$ (d) None of these

47. If a line makes an angle of $\pi/4$ with the positive directions of each of x- axis and y- axis, then the angle that the line makes with the positive direction of the z-axis is **[2007]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

48. If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for **[2007]**

(a) no value of θ
(b) exactly one value of θ
(c) exactly two values of θ
(d) more than two values of θ

49. If (2, 3, 5) is one end of a diameter of the sphere $x^2+y^2+z^2-6x-12y-2z+20=0$, then the cooordinates of the other end of the diameter are **[2007]**

(a) $(4,3,5)$ (b) $(4,3,-3)$
(c) $(4,9,-3)$ (d) $(4,-3,3)$.

50. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$. If the vectors $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals **[2007]**

(a) – 4 (b) – 2
(c) 0 (d) 1.

51. Let L be the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$. If L makes an angle α with the positive x-axis, then cos α equals **[2007]**

(a) 1 (b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

52. The vector $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$ lies in the plane of the vectors $\vec{b}=\hat{i}+\hat{j}$ and $\vec{c}=\hat{j}+\hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β? **[2008]**

(a) $\alpha=2, \beta=2$ (b) $\alpha=1, \beta=2$
(c) $\alpha=2, \beta=1$ (d) $\alpha=1, \beta=1$

53. The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is **[2008]**

(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) π

54. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

(a) $a=2, b=8$ (b) $a=4, b=6$
(c) $a=6, b=4$ (d) $a=8, b=2$

55. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to **[2008]**

(a) –5 (b) 5
(c) 2 (d) –2

56. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$. Then (α, β) equals **[2009]**

(a) $(-6, 7)$ (b) $(5, -15)$
(c) $(-5, 5)$ (d) $(6, -17)$

57. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are : **[2009]**

(a) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$ (b) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

(c) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$ (d) $6, -3, 2$

58. If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $[3\vec{u}\ p\vec{v}\ p\vec{\omega}]-[p\vec{v}\ \vec{\omega}\ q\vec{u}]-[2\vec{\omega}\ q\vec{v}\ q\vec{u}]=0$ holds for : **[2009]**

(a) exactly two values of (p, q)
(b) more than two but not all values of (p, q)
(c) all values of (p, q)
(d) exactly one value of (p, q)

59. Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}.\vec{b}=3$ **[2010]**

(a) $2\hat{i}-\hat{j}+2\hat{k}$ (b) $\hat{i}-\hat{j}-2\hat{k}$

(c) $\hat{i}+\hat{j}-2\hat{k}$ (d) $-\hat{i}+\hat{j}-2\hat{k}$

60. If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu)=$ **[2010]**

(a) $(2, -3)$ (b) $(-2, 3)$
(c) $(3, -2)$ (d) $(-3, 2)$

61. **Statement -1 :** The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x-y+z=5$.
Statement -2: The plane $x-y+z=5$ bisects the line segment joining A(3, 1, 6) and B(1, 3, 4). **[2010]**

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

62. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals **[2010]**

(a) 45° (b) 60° (c) 75° (d) 30°

63. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals **[2011]**

(a) $\frac{3}{2}$ (b) $\frac{2}{5}$

(c) $\frac{5}{3}$ (d) $\frac{2}{3}$

64. If $\vec{a}=\frac{1}{\sqrt{10}}(3\hat{i}+\hat{k})$ and $\vec{b}=\frac{1}{7}(2\hat{i}+3\hat{j}-6\hat{k})$, then the value of $(2\vec{a}-\vec{b})\left[(\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})\right]$ is **[2011]**

(a) –3 (b) 5 (c) 3 (d) –5

65. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$ and $\vec{a}.\vec{d}=0$. Then the vector $\vec{d}$ is equal to **[2011]**

(a) $\vec{c}+\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (b) $\vec{b}+\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

(c) $\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$ (d) $\vec{b}-\left(\frac{\vec{b}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{c}$

66. **Statement-1:** The point $A(1, 0, 7)$) is the mirror image of the point $B(1, 6, 3)$ in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ **[2011]**

Statement-2: The line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(a) Statement-1 is true, Statement-2 is true; Statement-2 is **not a** correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

67. Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is : **[2012]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

68. A equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is : **[2012]**

(a) $x - 2y + 2z - 3 = 0$ (b) $x - 2y + 2z + 1 = 0$
(c) $x - 2y + 2z - 1 = 0$ (d) $x - 2y + 2z + 5 = 0$

69. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to: **[2012]**

(a) -1 (b) $\frac{2}{9}$
(c) $\frac{9}{2}$ (d) 0

70. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}, \overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : **[2012]**

(a) $\vec{r} = 3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (b) $\vec{r} = -\vec{q} + \frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

(c) $\vec{r} = \vec{q} - \frac{(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (d) $\vec{r} = -3\vec{q} - \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

71. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is **[JEE M 2013]**

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$ (c) $\frac{7}{2}$ (d) $\frac{9}{2}$

72. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have **[JEE M 2013]**

(a) any value (b) exactly one value
(c) exactly two values (d) exactly three values

73. If the vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ and $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is **[JEE M 2013]**

(a) $\sqrt{18}$ (b) $\sqrt{72}$
(c) $\sqrt{33}$ (d) $\sqrt{45}$

74. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line: **[JEE M 2014]**

(a) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(b) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(c) $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(d) $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

75. The angle between the lines whose direction cosines satisfy the equations $l + m + n = 0$ and $l^2 = m^2 + n^2$ is **[JEE M 2014]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

76. If $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = \lambda\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$ then λ is equal to **[JEE M 2014]**

(a) 0 (b) 1 (c) 2 (d) 3

77. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is : **[JEE M 2015]**

(a) $\frac{2}{3}$ (b) $\frac{-2\sqrt{3}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{-\sqrt{2}}{3}$

78. The equation of the plane containing the line $2x - 5y + z = 3$; $x + y + 4z = 5$, and parallel to the plane, $x + 3y + 6z = 1$, is : **[JEE M 2015]**

(a) $x + 3y + 6z = 7$ (b) $2x + 6y + 12z = -13$
(c) $2x + 6y + 12z = 13$ (d) $x + 3y + 6z = -7$

79. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 16$, is **[JEE M 2015]**

(a) $3\sqrt{21}$ (b) 13 (c) $2\sqrt{14}$ (d) 8

80. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to : **[JEE M 2016]**

(a) 5 (b) 2 (c) 26 (d) 18

81. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\sqrt{3}}{2}\left(\vec{b} + \vec{c}\right)$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is: **[JEE M 2016]**

(a) $\frac{2\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

82. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along the line $x = y = z$ is : **[JEE M 2016]**

(a) $\frac{10}{\sqrt{3}}$ (b) $\frac{20}{3}$

(c) $3\sqrt{10}$ (d) $10\sqrt{3}$

GP_3480

20 Vector Algebra and Three Dimensional Geometry

Section-A : JEE Advanced/ IIT-JEE

A

1. $5\sqrt{2}$ **2.** $\pm\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}$ **3.** $\sqrt{13}$ **4.** orthocentre **5.** -1 **6.** 0

7. $\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$ **8.** 1 **9.** $2\hat{i}-\hat{j}$ **10.** $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$, $\vec{a}-\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

11. $\frac{5\hat{i}+2\hat{j}+2\hat{k}}{3}$ **12.** $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$ or $\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$ **13.** $-\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$ **14.** $\frac{\pi}{4}$ or $\frac{3\pi}{4}$ **15.** $\vec{a}$

16. 6

B

1. T **2.** T **3.** T **4.** F

C

1. (a) **2.** (d) **3.** (d) **4.** (a) **5.** (d) **6.** (b) **7.** (a)
8. (b) **9.** (a) **10.** (a) **11.** (b) **12.** (d) **13.** (b) **14.** (a)
15. (b) **16.** (a) **17.** (a) **18.** (c) **19.** (b) **20.** (b) **21.** (c)
22. (a) **23.** (c) **24.** (c) **25.** (b) **26.** (c) **27.** (d) **28.** (b)
29. (d) **30.** (a) **31.** (c) **32.** (b) **33.** (a) **34.** (a) **35.** (a)
36. (c) **37.** (c) **38.** (a) **39.** (c) **40.** (a) **41.** (b) **42.** (c)
43. (a) **44.** (a) **45.** (c) **46.** (c)

D

1. (c) **2.** (b) **3.** (a, c) **4.** (a, c, d) **5.** (d) **6.** (c)
7. (a, c) **8.** (a, c) **9.** (b, d) **10.** (a, d) **11.** (b, c) **12.** (b, d) **13.** (a, d)
14. (a, b, c) **15.** (c) **16.** (b, d) **17.** (a, b) **18.** (a, c, d) **19.** (b, c, d) **20.** (b, c)

E

3. $\lambda = 0, -1$ **4.** $A_2\hat{i}-A_1\hat{j}+A_3\hat{k}$ **5.** $\frac{146}{17}$ **10.** $-\hat{i}-8\hat{j}+2\hat{k}$ **11.** $-\frac{4}{3}<c<0$

12. $8:3$ **14.** $(-1, 3, 3)$ or $(3, -1, -1)$

20. $\vec{v_1}=2\hat{i}$; $\vec{v_2}=-\hat{i}\pm\hat{j}$; $\vec{v_3}=3\hat{i}\pm2\hat{j}\pm4\hat{k}$ are some possible values. **23.** **(i)** $x+y-2z=3$ **(ii)** $Q(6, 5, -2)$

26. $x+y+z=3$; $\frac{9}{2}$ cubic units **29.** $62x+29y+19z-105=0$ **30.** $\hat{\omega}=\hat{v}-2(\hat{a}\cdot\hat{v})\hat{a}$

F

1. (A) → s; (B) → p; (C) → r, q; (D) → s **2.** (A) → r; (B) → q; (C) → p; (D) → s
3. (A) → q, s; (B) → p, r, s,t; (C) → t; (D) → r **4.** (A) → p; (B) → q, s; (C) → q, r, s, t; (D) → r
5. (A) → t; (B) → p, r; (C) → q, s; (D) → r **6.** (A) → q; (B) → p; (C) → s; (D) → t
7. **(c)** **8.** **(a)**
9. **(a)** **10.** (A) → q; (B) → p, q; (C) → p, q, s, t; (D) → q, t
11. (A) → p, r, s; (B) → p; (C) → p, q; (D) → s, t

G

1. (b) **2.** (d) **3.** (c)

H

1. (d) **2.** (c) **3.** (d)

I

1. 5 **2.** 6 **3.** 9 **4.** 3 **5.** 5 **6.** 5 **7.** 4
8. 9

Section-B : JEE Main/ AIEEE

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (b)
8. (c)	**9.** (b)	**10.** (a)	**11.** (d)	**12.** (d)	**13.** (d)	**14.** (a)
15. (d)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (none)	**21.** (c)
22. (a)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (c)	**28.** (d)
29. (c)	**30.** (c)	**31.** (a)	**32.** (a)	**33.** (a)	**34.** (b)	**35.** (c)
36. (b)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (b)	**41.** (d)	**42.** (b)
43. (d)	**44.** (a)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (b)	**49.** (c)
50. (b)	**51.** (c)	**52.** (d)	**53.** (d)	**54.** (c)	**55.** (a)	**56.** (a)
57. (b)	**58.** (d)	**59.** (d)	**60.** (d)	**61.** (a)	**62.** (b)	**63.** (d)
64. (d)	**65.** (c)	**66.** (a)	**67.** (c)	**68.** (a)	**69.** (c)	**70.** (b)
71. (c)	**72.** (c)	**73.** (c)	**74.** (c)	**75.** (c)	**76.** (b)	**77.** (c)
78. (a)	**79.** (b)	**80.** (b)	**81.** (b)	**82.** (d)		

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. Given that $|\vec{A}|=3$; $|\vec{B}|=4$; $|\vec{C}|=5$

$\vec{A}\perp(\vec{B}+\vec{C})\Rightarrow\vec{A}.(\vec{B}+\vec{C})=0\ \Rightarrow\vec{A}.\vec{B}+\vec{A}.\vec{C}=0$ ...(1)

$\vec{B}\perp(\vec{C}+\vec{A})\Rightarrow\vec{B}.(\vec{C}+\vec{A})=0\ \Rightarrow\vec{B}.\vec{C}+\vec{B}.\vec{A}=0$ ...(2)

$\vec{C}\perp(\vec{A}+\vec{B})\Rightarrow\vec{C}.(\vec{A}+\vec{B})=0\ \Rightarrow\vec{C}.\vec{A}+\vec{C}.\vec{B}=0$ ...(3)

Adding (1), (2) and (3) we get

$2\left(\vec{A}\,\vec{B}+\vec{B}\,\vec{C}+\vec{C}\,\vec{A}\right)=0$ ...(4)

Now, $|\vec{A}+\vec{B}+\vec{C}|^2=\left(\vec{A}+\vec{B}+\vec{C}\right).\left(\vec{A}+\vec{B}+\vec{C}\right)$

$=\vec{A}.\vec{A}+\vec{B}.\vec{B}+\vec{C}.\vec{C}+2\vec{A}.\vec{B}+2\vec{B}.\vec{C}+2\vec{C}.\vec{A}$

$=|\vec{A}|^2+|\vec{B}|^2+|\vec{C}|^2+2\left(\vec{A}.\vec{B}+\vec{B}.\vec{C}+\vec{C}.\vec{A}\right)$

$=9+16+25+0$ (using equation 4)

$=50$ $\quad\therefore\ |\vec{A}+\vec{B}+\vec{C}|=5\sqrt{2}$

2. Required unit vector, $\hat{n}=\pm\dfrac{\overrightarrow{PQ}\times\overrightarrow{PR}}{|\overrightarrow{PQ}\times\overrightarrow{PR}|}$

$\overrightarrow{PQ}=\hat{i}+\hat{j}-3\hat{k}$; $\overrightarrow{PR}=-\hat{i}+3\hat{j}-\hat{k}$

$$\therefore\ \overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-3\\-1&3&-1\end{vmatrix}$$

$=(-1+9)\hat{i}+(3+1)\hat{j}+(3+1)\hat{k}\ =8\hat{i}+4\hat{j}+4\hat{k}$

$|\overrightarrow{PQ}\times\overrightarrow{PR}|=\sqrt{64+16+16}=\sqrt{96}=4\sqrt{6}$

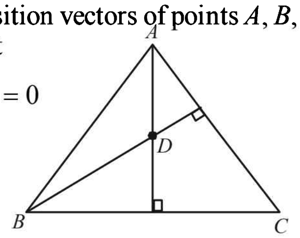
$$\hat{n}=\pm\left(\frac{8\hat{i}+4\hat{j}+4\hat{k}}{4\sqrt{6}}\right)=\pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

3. Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{BA}\times\overrightarrow{BC}|$

$\overrightarrow{BA}=-\hat{i}-2\hat{j}+3\hat{k}$, $\overrightarrow{BC}=\hat{i}-2\hat{j}+3\hat{k}$

$$\therefore\Delta=\frac{1}{2}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&-2&3\\1&-2&3\end{vmatrix}=\frac{1}{2}|6\hat{j}+4\hat{k}|=|3\hat{j}+2\hat{k}|=\sqrt{9+4}=\sqrt{13}$$

4. Given that $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are position vectors of points A, B, C and D respectively, such that



$\left(\vec{a}-\vec{d}\right).(\vec{b}-\vec{c})=(\vec{b}-\vec{d}).(\vec{c}-\vec{a})=0$

$\Rightarrow\ \overrightarrow{DA}.\overrightarrow{CB}=\overrightarrow{DB}.\overrightarrow{AC}=0$

$\Rightarrow\overrightarrow{DA}\perp\overrightarrow{CB}$ and $\overrightarrow{DB}\perp\overrightarrow{AC}$

Clearly D is orthocentre of $\Delta\,ABC$

5. Given that $\begin{vmatrix}a&a^2&1+a^3\\b&b^2&1+b^3\\c&c^2&1+c^3\end{vmatrix}=0$

$$\Rightarrow\begin{vmatrix}a&a^2&1\\b&b^2&1\\c&c^2&1\end{vmatrix}+abc\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}=0$$

Operating $C_2\leftrightarrow C_3$ and then $C_1\leftrightarrow C_2$ in first determinant

$$\Rightarrow\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}+abc\begin{vmatrix}1&a&a^2\\1&b&b^2\\1&c&c^2\end{vmatrix}=0$$

$$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \text{either } 1 + abc = 0 \text{ or } \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

Also given that the vectors $\vec{A},\vec{B},\vec{C}$ are noncoplanar

i.e., $[\vec{A}\vec{B}\vec{C}] \neq 0$ where $\vec{A} = \hat{i} + a\hat{j} + a^2\hat{k}$

$\vec{B} = \hat{i} + b\hat{j} + b^2\hat{k}$, $\vec{C} = \hat{i} + c\hat{j} + c^2\hat{k}$ $\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$

$\therefore$ We must have $1 + abc = 0 \Rightarrow abc = -1$

6. As given that $\vec{A},\vec{B},\vec{C}$ are three noncoplanar vectors, therefore, $[\vec{A}\vec{B}\vec{C}] \neq 0$

Also by the property of scalar triple product we have

$\vec{A}.(\vec{B}\times\vec{C}) = [\vec{A}\vec{B}\vec{C}]$, $\vec{B}.(\vec{A}\times\vec{C}) = -[\vec{A}\vec{B}\vec{C}]$

$\vec{C}\times(\vec{A}.\vec{B}) = [\vec{A}\vec{B}\vec{C}]$, $\vec{C}.(\vec{A}\times\vec{B}) = [\vec{A}\vec{B}\vec{C}]$

$$\therefore \frac{\vec{A}.(\vec{B}\times\vec{C})}{(\vec{C}\times\vec{A}).\vec{B}} + \frac{\vec{B}.(\vec{A}\times\vec{C})}{(\vec{C}\times\vec{A}).\vec{B}} = \frac{[\vec{A}\vec{B}\vec{C}]}{[\vec{A}\vec{B}\vec{C}]} + \frac{-[\vec{A}\vec{B}\vec{C}]}{[\vec{A}\vec{B}\vec{C}]} = 0$$

7. Given $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{C} = \hat{j} - \hat{k}$

Let $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$$ATQ,\ \vec{A}\times\vec{B} = \vec{C} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$$

$$\Rightarrow (z-y)\,\hat{i} + (x-z)\,\hat{j} + (y-x)\,\hat{k} = \hat{j} - \hat{k}$$

$$\Rightarrow \begin{matrix} z-y=0 \\ x-z=1 \\ y-x=-1 \end{matrix} \Rightarrow \left.\begin{matrix} y=z \\ x=1+z \end{matrix}\right\} \quad ...(1)$$

Also, $\vec{A}.\vec{B} = 3 \Rightarrow x + y + z = 3$ ...(2)

Using equations (1) and (2) we get

$1 + z + z + z = 3$

$\Rightarrow z = 2/3 \Rightarrow y = 2/3, x = 5/3$

$$\therefore \vec{B} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

8. Given that the vectors $\hat{u} = a\hat{i} + \hat{j} + \hat{k}$, $\hat{v} = \hat{i} + b\hat{j} + \hat{k}$ and $\hat{w} = \hat{i} + \hat{j} + c\hat{k}$ where a $\neq$ b $\neq$ c $\neq$1 are coplanar

$$\therefore [\vec{u}\,\vec{v}\,\vec{w}] = 0 \Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Oeprating $C_1 - C_2$, $C_2 - C_3$

$$\begin{vmatrix} a-1 & 0 & 1 \\ 1-b & b-1 & 1 \\ 0 & 1-c & c \end{vmatrix} = 0$$

Taking $(1-a)$, $(1-b)$, $(1-c)$ common from R_1, R_2 and R_3 respectively.

$$\Rightarrow (1-a)(1-b)(1-c)\begin{vmatrix} -1 & 0 & \frac{1}{1-a} \\ 1 & -1 & \frac{1}{1-b} \\ 0 & 1 & \frac{c}{1-c} \end{vmatrix} = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[-\left\{\frac{-c}{1-c} - \frac{1}{1-b}\right\} + \frac{1}{1-a}(1-0)\right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[\frac{1}{1-a} + \frac{1}{1-b} + \frac{c}{1-c}\right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[\frac{1}{1-a} + \frac{1}{1-b} - \frac{(1-c)-1}{1-c}\right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)\left[\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1\right] = 0$$

But $a \neq b \neq c \neq 1$ (given)

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1 = 0 \Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

9. Let $\vec{c} = \alpha\hat{i} + \beta\hat{j}$

As $\vec{b} \perp \vec{c}$ (given) $\quad \therefore \vec{b}.\vec{c} = 0$

$\Rightarrow (4\hat{i} + 3\hat{j}).(\alpha\hat{i} + \beta\hat{j}) = 0 \Rightarrow 4\alpha + 3\beta = 0$

$$\Rightarrow \alpha = -\frac{3\beta}{4} \Rightarrow \frac{\alpha}{+3} = \frac{\beta}{-4} = \lambda$$

$\Rightarrow \alpha = +3\lambda, \beta = -4\lambda$ ...(1)

Now, let $\vec{a} = x\hat{i} + y\hat{j}$ be the required vectors.

Then as per question

$$\text{Projection of } \vec{a} \text{ along } \vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|} \Rightarrow \frac{4x+3y}{\sqrt{4^2+3^2}} = 1$$

$\Rightarrow 4x + 3y = 5$ ...(2)

Also, projection of $\vec{a}$ along $\vec{c} = 2$

$$\Rightarrow \frac{\vec{a}.\vec{c}}{|\vec{c}|} = 2 \Rightarrow \frac{\alpha x + \beta y}{\sqrt{\alpha^2+\beta^2}} = 2 \Rightarrow \frac{3\lambda x - 4\lambda y}{\sqrt{(3\lambda)^2 + (-4\lambda)^2}} = 2$$

$\Rightarrow 3\lambda x - 4\lambda y = 10\lambda$

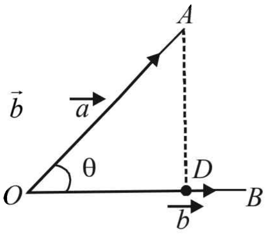
$\Rightarrow 3x - 4y = 10$ ...(3)

Solving (2) and (3), we get $x = 2, y = -1$

$\therefore$ The required vector is $2\hat{i} - \hat{j}$

10. Component of $\vec{a}$ along $\vec{b} = \overrightarrow{OD} = OA\cos\theta\,.\hat{b}$

$$= \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|}\right)\frac{\vec{b}}{|\vec{b}|} = \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$$



Component of $\vec{a}$ perpendicular to $\vec{b}$

$$= \overrightarrow{DA} = \vec{a} - \overrightarrow{OD} = \vec{a} - \left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$$

11. See the solution to Q-7

12. Let $x\hat{i}+y\hat{j}+z\hat{k}$ be a unit vector, coplanar with $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ and also perpendicular to $\hat{i}+\hat{j}+\hat{k}$

Then, $\begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow -3x+y+z=0$ .....(i)

and $x+y+z=0$ .....(ii)

Solving the above by cross multiplication method, we get

$$\frac{x}{0}=\frac{y}{4}=\frac{z}{-4} \text{ or } \frac{x}{0}=\frac{y}{1}=\frac{z}{-1}=\lambda(say)$$

$\Rightarrow x=0, y=\lambda, z=-\lambda$

As $x\hat{i}+y\hat{j}+z\hat{k}$ is a unit vector, therefore

$$0+\lambda^2+\lambda^2=1 \Rightarrow \lambda^2=\frac{1}{2} \Rightarrow \lambda=\pm\frac{1}{\sqrt{2}}$$

$\therefore$ The required vector is $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$ or $\frac{-\hat{j}+\hat{k}}{\sqrt{2}}$

13. We have position vectors of points $P\left(\hat{i}-\hat{j}+2\hat{k}\right)$, $Q\left(2\hat{i}-\hat{k}\right)$, $R\left(2\hat{j}+\hat{k}\right)$

$\therefore \overrightarrow{QP} = (\hat{i}-\hat{j}+2\hat{k})-(2\hat{i}-\hat{k}) = -\hat{i}-\hat{j}+3\hat{k}$

$\therefore \overrightarrow{QR} = 2\hat{j}+\hat{k}-2\hat{i}+\hat{k} = -2\hat{i}+2\hat{j}+2\hat{k}$

Now any vector perpendicular to the plane formed by pts PQR is given by $\overrightarrow{QP}\times\overrightarrow{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 3 \\ -2 & 2 & 2 \end{vmatrix} = -8\hat{i}-4\hat{j}-4\hat{k}$

$\therefore$ Unit vector normal to plane $= \pm\left(\frac{-8\hat{i}-4\hat{j}-4\hat{k}}{\sqrt{64+16+16}}\right)$

$$= \pm\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

14. Eq^n of plane containing vectors $\hat{i}$ and $\hat{i}+\hat{j}$ is

$$\left[\vec{r}-\hat{i}\;\;\hat{i}\;\;\hat{i}+\hat{j}\right]=0 \Rightarrow \begin{vmatrix} x-1 & y & z \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix}=0$$

$\Rightarrow z=0$ ....(1)

Similarly, eq^n of plane containing vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ is

$$[\hat{r}-(\hat{i}-\hat{j})\;\;\hat{i}-\hat{j}\;\;\hat{i}+\hat{k}]=0 \Rightarrow \begin{vmatrix} x-1 & y+1 & z \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix}=0$$

$\Rightarrow (x-1)(-1-0)-(y+1)(1-0)+z(0+1)=0$

$\Rightarrow -x+1-y-1+z=0$

$\Rightarrow x+y-z=0$ ....(2)

Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

Since $\vec{a}$ is parallel to (1) and (2)

$a_3=0$ and $a_1+a_2-a_3=0 \Rightarrow a_1=-a_2, a_3=0$

$\therefore$ a vector in direction of $\vec{a}$ is $\hat{i}-\hat{j}$

Now if θ is the angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ then

$$\cos\theta = \pm\frac{1.1+(-1)(-2)}{\sqrt{1+1}\sqrt{1+4+4}} = \pm\frac{3}{\sqrt{2}.3}$$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \pi/4$ or $3\pi/4$

15. Let us consider $\vec{b}=\hat{i}$ and $\vec{c}=\hat{j}$ then $\vec{b}\times\vec{c}=\hat{k}$

Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

Then, $\left(\vec{a}\vec{b}\right)\vec{b}+(\vec{a}.\vec{c})\vec{c}+\frac{\vec{a}.\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|}(\vec{b}\times\vec{c}) = x\hat{i}+y\hat{j}+z\hat{k} = \hat{a}$

16. q = area of parallelogram with $\overline{OA}$ and $\overline{OC}$ as adjacent sides $= \left|\overline{OA}\times\overline{OC}\right| = \left|\vec{a}\times\vec{b}\right|$

and p = area of quadrilateral $OABC$

$$= \frac{1}{2}\left|\overline{OA}\times\overline{OB}\right| + \frac{1}{2}\left|\overline{OB}\times\overline{OC}\right|$$

$$= \frac{1}{2}\left|\vec{a}\times\left(\overline{10a}+\overline{2b}\right)\right| + \frac{1}{2}\left|\left(\overline{10a}+\overline{2b}\right)\times\vec{b}\right|$$

$$= \left|\vec{a}\times\vec{b}\right| + 5\left|\vec{a}\times\vec{b}\right| = 6\left|\vec{a}\times\vec{b}\right| \quad \therefore\; p = 6q \Rightarrow k = 6$$

B. True/False

1. $\vec{A}$, $\vec{B}$, $\vec{C}$ are three unit vectors such that

$\vec{A}.\vec{B}=\vec{A}.\vec{C}=0$ ...(1)

and angle between $\vec{B}$ and $\vec{C}$ is $\pi/6$.

Now eq. (1) shows that $\vec{A}$ is perpendicular to both $\vec{B}$ and $\vec{C}$.

$\therefore \vec{B}\times\vec{C} \,||\, \vec{A} \Rightarrow \vec{B}\times\vec{C}=\lambda\vec{A}$ where λ is any scalar.

$\Rightarrow |\vec{B}\times\vec{C}| = |\lambda\vec{A}| \Rightarrow \sin\pi/6 = \pm\lambda$

(as $\pi/6$ is the angle between $\vec{B}$ & $\vec{C}$)

$\Rightarrow \lambda = \pm\frac{1}{2} \Rightarrow \vec{B}\times\vec{C} = \pm\frac{1}{2}\vec{A} \Rightarrow \vec{A} = \pm 2(\vec{B}\times\vec{C})$

$\therefore$ Given statement is true.

2. $\vec{X}.\vec{A} = 0 \Rightarrow$ either $\vec{A} = 0$ or $\vec{X}\perp\vec{A}$

$\vec{X}.\vec{B} = 0 \Rightarrow$ either $\vec{B} = 0$ or $\vec{X}\perp\vec{B}$

$\vec{X}.\vec{C} = 0 \Rightarrow$ either $\vec{C} = 0$ or $\vec{X}\perp\vec{C}$

In any of three cases,

if $\vec{A}$ or $\vec{B}$ or $\vec{C} = 0 \Rightarrow [\vec{A}\vec{B}\vec{C}] = 0$

Otherwise if $\vec{X}\perp\vec{A}, \vec{X}\perp\vec{B}, \vec{X}\perp\vec{C}$ then $\vec{A},\vec{B},\vec{C}$ are coplanar $\Rightarrow [\vec{A}\vec{B}\vec{C}] = 0$

$\therefore$ Given statement is true.

3. Let position vectors of pts A, B and C be $\vec{a}+\vec{b}$, $\vec{a}-\vec{b}$ and $\vec{a}+k\vec{b}$ respectively.

Then, $\overrightarrow{AB}$ = p.v. of B – p.v of A $= (\vec{a}-\vec{b})-(\vec{a}+\vec{b}) = -2\vec{b}$

Similarly, $\overrightarrow{BC} = \vec{a}+k\vec{b}-\vec{a}+\vec{b} = (k+1)\vec{b}$

Clearly $\overrightarrow{AB} \parallel \overrightarrow{BC}$ $\forall k \in R$

$\Rightarrow A, B, C$ are collinear $\forall k \in R$

$\therefore$ Statement is true.

4. For any three vectors $\vec{a},\vec{b}$ and $\vec{c}$, we have

L.H.S. $= (\vec{a}-\vec{b}).(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})$

$= (\vec{a}-\vec{b}).(\vec{b}\times\vec{c}-\vec{b}\times\vec{a}-\vec{c}\times\vec{c}+\vec{c}\times\vec{a})$

$= (\vec{a}-\vec{b}).(\vec{b}\times\vec{c}+\vec{a}\times\vec{b}+\vec{c}\times\vec{a})$

$= \vec{a}.(\vec{b}\times\vec{c})+\vec{a}.(\vec{a}\times\vec{b})+\vec{a}.(\vec{c}\times\vec{a})$
$\quad -\vec{b}.(\vec{b}\times\vec{c})-\vec{b}.(\vec{a}\times\vec{b})-\vec{b}.(\vec{c}\times\vec{a})$

$= [\vec{a}\vec{b}\vec{c}]-[\vec{a}\vec{b}\vec{c}] = 0 \neq$ R.H.S.

$\therefore$ The given statement is false.

C. MCQs with ONE Correct Answer

1. (a) $\vec{A}.(\vec{B}+\vec{C})\times(\vec{A}+\vec{B}+\vec{C})$

$= \vec{A}.[\vec{B}\times\vec{A}+\vec{B}\times\vec{B}+\vec{B}\times\vec{C}+\vec{C}\times\vec{A}+\vec{C}\times\vec{B}+\vec{C}\times\vec{C}]$

$= \vec{A}.\vec{B}\times\vec{A}+\vec{A}.\vec{B}\times\vec{C}+\vec{A}.\vec{C}\times\vec{A}+\vec{A}.\vec{C}\times\vec{B}$

(Using $\vec{a}\times\vec{a} = 0$)

$= 0+[\vec{A}\vec{B}\vec{C}]+0+[\vec{A}\vec{C}\vec{B}]$

(as $[\vec{a}\vec{b}\vec{c}] = 0$ if any two vector are equal out of $\vec{a},\vec{b},\vec{c}$)

$= [\vec{A}\vec{B}\vec{C}]-[\vec{A}\vec{B}\vec{C}]$ [Using $[\vec{a}\vec{b}\vec{c}] = -[\vec{a}\vec{c}\vec{b}]$]

$= 0$

2. (d) $|(\vec{a}\times\vec{b}).\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$

$\Rightarrow |\vec{a}||\vec{b}|\,|\sin\theta\,\hat{n}.\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$

where θ is angle between $\vec{a}$ and $\vec{b}$.

$\Rightarrow |\vec{a}||\vec{b}||\vec{c}|\,|\sin\theta\cos\alpha|$

$= |\vec{a}||\vec{b}||\vec{c}|$ where α is angle between $\vec{c}$ and $\hat{n}$.

$\Rightarrow |\sin\theta||\cos\alpha| = 1 \Rightarrow \theta = \pi/2$ and $\alpha = 0$

$\Rightarrow \vec{a}\perp\vec{b}$ and $\vec{c}\parallel\hat{n} \Rightarrow \vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

3. (d) Vol. of parallelopiped $= [\vec{a}\,\vec{b}\,\vec{c}]$

$$= \begin{vmatrix} 2 & -2 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} = 2(-1)+2(-1+3) = 2$$

4. (a) Three pts A, B, C are collinear if $\overrightarrow{AB} \parallel \overrightarrow{AC}$

$\overrightarrow{AB} = -20\hat{i}-11\hat{j}$; $\overrightarrow{AC} = (a-60)\hat{i}-55\hat{j}$

$\overrightarrow{AB}\parallel\overrightarrow{AC} \Rightarrow \frac{a-60}{-20} = \frac{-55}{-11} \Rightarrow a = -40$

5. (d) Given that $\vec{a},\vec{b},\vec{c}$ are non coplanar

$\therefore [\vec{a}\,\vec{b}\,\vec{c}] \neq 0$

Also $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ ..(1)

Now, $(\vec{a}+\vec{b}).\vec{p}+(\vec{b}+\vec{c}).\vec{q}+(\vec{c}+\vec{a}).\vec{r}$

$= (\vec{a}+\vec{b}).\frac{\vec{b}\times\vec{c}}{[\vec{a}\vec{b}\vec{c}]}+(\vec{b}+\vec{c}).\frac{\vec{c}\times\vec{a}}{[\vec{a}\vec{b}\vec{c}]}+(\vec{c}+\vec{a}).\frac{\vec{a}\times\vec{b}}{[\vec{a}\vec{b}\vec{c}]}$

$= \frac{\vec{a}.\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}+\frac{\vec{b}.\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}+\frac{\vec{c}.\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$

[Using $\vec{b}.\vec{b}\times\vec{c} = \vec{c}.\vec{c}\times\vec{a} = \vec{a}.\vec{a}\times\vec{b} = 0$]

$= \frac{[\vec{a}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]}+\frac{[\vec{a}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]}+\frac{[\vec{a}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]} = 1+1+1 = 3$

6. (b) a, b, c are distinct non negative numbers and the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a & c-a \\ 1 & 0 & 0 \\ c & c & b-c \end{vmatrix}$$

Operating $C_3 \to C_3 - C_1$

Expanding along R_2, we get

$$-\begin{vmatrix} a & c-a \\ c & b-c \end{vmatrix} = c(c-a)-a(b-c) = 0$$

$\Rightarrow c^2 - ac - ab + ac = 0$

$\Rightarrow c^2 = ab \Rightarrow a, c, b$ are in G.P.

$\therefore$ c is the G.M. of a and b.

7. (a) We have $\overrightarrow{OR} = \frac{3\vec{p}+2\vec{q}}{3+2} = \frac{1}{2}(3\vec{p}+2\vec{q})$

[Internal divison]

and $\overrightarrow{OS} = \frac{3\vec{p}-2\vec{q}}{3-2} = 3\vec{p}-2\vec{q}$

[external divison]

Given $\overrightarrow{OR} \perp \overrightarrow{OS} \Rightarrow \overrightarrow{OR}.\overrightarrow{OS} = 0$

$\Rightarrow \frac{1}{5}[3\vec{p}+2\vec{q}].(3\vec{p}-2\vec{q}) = 0$

$\Rightarrow 9|\vec{p}|^2 = 4|\vec{q}|^2 \Rightarrow 9p^2 = 4q^2$

8. **(b)** Let the given position vectors be of point A, B and C respectively, then

$|\overrightarrow{AB}| = \sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$

$|\overrightarrow{BC}| = \sqrt{(\gamma-\beta)^2+(\alpha-\gamma)^2+(\alpha-\beta)^2}$

$|\overrightarrow{CA}| = \sqrt{(\alpha-\gamma)^2+(\beta-\alpha)^2+(\gamma-\beta)^2}$

$\because |\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CA}|$

$\Rightarrow \Delta ABC$ is an equilateral Δ.

9. **(a)** Let $\vec{d} = x\hat{i}+y\hat{j}+z\hat{k}$

where $x^2+y^2+z^2 = 1$ ....(1)

($\vec{d}$ being unit vector) $\therefore \vec{a}.\vec{d} = 0$

$\Rightarrow x - y = 0 \Rightarrow x = y$ ...(2)

$$\left[\vec{b}\ \vec{c}\ \vec{d}\right] = 0 \Rightarrow \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ x & y & z \end{vmatrix} = 0$$

$\Rightarrow x + y + z = 0$

$\Rightarrow 2x + z = 0$ (using (2))

$\Rightarrow z = -2x$ ...(3)

From (1), (2) and (3)

$x^2 + x^2 + 4x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{6}}$

$$\therefore d = \pm\left(\frac{1}{\sqrt{6}}\vec{i}+\frac{1}{\sqrt{6}}\vec{j}-\frac{2}{\sqrt{6}}\vec{k}\right) = \pm\left(\frac{\vec{i}+\vec{j}-2\vec{k}}{\sqrt{6}}\right)$$

10. **(a)** Since $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$

$\therefore (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{1}{\sqrt{2}}\vec{b}+\frac{1}{\sqrt{2}}\vec{c} \Rightarrow \vec{a}.\vec{c} = \frac{1}{\sqrt{2}}$

[$\because \vec{b}$ and $\vec{c}$ are non-coplanar]

and $\vec{a}.\vec{b} = -\frac{1}{\sqrt{2}} \Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\frac{3\pi}{4} = \cos\theta \Rightarrow \theta = 3\pi/4$

11. **(b)** $\because \vec{u}+\vec{v}+\vec{w} = 0 \quad \therefore |\vec{u}+\vec{v}+\vec{w}|^2 = 0$

$\Rightarrow |\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2+2(\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}) = 0$

$\Rightarrow 9+16+25+2(\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}) = 0$

$\Rightarrow (\vec{u}.\vec{v}+\vec{v}.\vec{w}+\vec{w}.\vec{u}) = -25$

12. **(d)** $(\vec{a}+\vec{b}+\vec{c}).[(\vec{a}+\vec{b})\times(\vec{a}+\vec{c})]$

$= (\vec{a}+\vec{b}+\vec{c}).[\vec{a}\times\vec{a}+\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}]$

$= (\vec{a}+\vec{b}+\vec{c}).[\vec{a}\times\vec{c}+\vec{b}\times\vec{a}+\vec{b}\times\vec{c}]$ [$\because \vec{a}\times\vec{a} = 0$]

$= \vec{a}.\vec{a}\times\vec{c}+\vec{a}.\vec{b}\times\vec{a}+\vec{a}.\vec{b}\times\vec{c}+\vec{b}.\vec{a}\times\vec{c}$

$+\vec{b}.\vec{b}\times\vec{a}+\vec{b}.\vec{b}\times\vec{c}+\vec{c}.\vec{a}\times\vec{c}+\vec{c}.\vec{b}\times\vec{a}+\vec{c}.\vec{b}\times\vec{c}$

$= [\vec{a}\vec{b}\vec{c}]-[\vec{a}\vec{b}\vec{c}]-[\vec{a}\vec{b}\vec{c}] = -[\vec{a}\vec{b}\vec{c}]$

13. **(b)** $|(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\sin 30°$

$= \frac{1}{2}|\vec{a}\times\vec{b}||\vec{c}|$ ...(1)

We have, $\vec{a} = 2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}$

$\Rightarrow \vec{a}\times\vec{b} = 2\hat{i}-2\hat{j}+\hat{k} \Rightarrow |\vec{a}\times\vec{b}| = \sqrt{9} = 3$

Also given $|\vec{c}-\vec{a}| = 2\sqrt{2}$

$\Rightarrow |\vec{c}-\vec{a}|^2 = 8 \Rightarrow (\vec{c}-\vec{a}).(\vec{c}-\vec{a}) = 8$

$\Rightarrow |\vec{c}|^2 - \vec{c}.\vec{a} - \vec{a}.\vec{c} + |\vec{a}|^2 = 8$

As $|\vec{a}| = 3$ and $\vec{a}.\vec{c} = |\vec{c}|$, we get

$|\vec{c}|^2 - 2|\vec{c}|+1 = 0 \quad (|\vec{c}|-1)^2 = 0 \Rightarrow |\vec{c}| = 1$

Substituting values of $|\vec{a}\times\vec{b}|$ and $|\vec{c}|$ in (1), we get

$|(\vec{a}\times\vec{b})\times\vec{c}| = \frac{1}{2}\times 3\times 1 = \frac{3}{2}$

14. **(a)** As c is coplanar with a and b, we take,

$c = \alpha a + \beta b$ ...(1)

where α, β are scalars.

As c is perpendicular to a, $c.a = 0$

$\therefore$ From (1) we get, $0 = \alpha\, a.a + \beta\, b.a$

$\Rightarrow 0 = \alpha(6)+\beta(2+2-1) = 3(2\alpha+\beta) \Rightarrow \beta = -2\alpha.$

Thus, $c = \alpha(a-2b) = \alpha(-3j+3k) = 3\alpha(-j+k)$

$\Rightarrow |c|^2 = 9\alpha^2(1+1) = 18\alpha^2 \Rightarrow 1 = 18\alpha^2$

$\Rightarrow \alpha = \pm\frac{1}{3\sqrt{2}} \qquad \therefore c = \pm\frac{1}{\sqrt{2}}(-j+k).$

Thus, we may take $c = \frac{1}{\sqrt{2}}(-j+k).$

15. **(b)** Given $\vec{a}+\vec{b}+\vec{c} = 0$ (by triangle law)

$\therefore \vec{a}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\times\vec{0} = \vec{0}$

$\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c} = \vec{0}$

$\Rightarrow \vec{a}\times\vec{b} = \vec{c}\times\vec{a}$ [$\because \vec{a}\times\vec{a} = 0$]

Similarly, $\vec{a}\times\vec{b} = \vec{b}\times\vec{c}$;

Therefore $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$.

16. **(a)** Given that $\vec{a},\vec{b},\vec{c},\vec{d}$ are vectors such that

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = 0$ ...(1)

P_1 is the plane determined by vectors $\vec{a}$ and $\vec{b}$

$\therefore$ Normal vectors $\overrightarrow{n_1}$ to P_1 will be given by $\overrightarrow{n_1} = \vec{a}\times\vec{b}$

Similarly, P_2 is the plane determined by vectors $\vec{c}$ and $\vec{d}$

$\therefore$ Normal vectors $\overrightarrow{n_2}$ to P_2 will be given by $\overrightarrow{n_2} = \vec{c}\times\vec{d}$

Substituting the values of $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ in eq^n (1)

We get, $\vec{n_1} \times \vec{n_2} = 0 \Rightarrow \vec{n_1} \parallel \vec{n_2}$
and hence the planes will also be parallel to each other. Thus angle between the planes = 0.

17. (a) $\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors, $2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}$ and $2\vec{c}-\vec{a}$ are also coplanar vectors. being linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$.

Thus, $[2\vec{a}-\vec{b} \quad 2\vec{b}-\vec{c} \quad 2\vec{c}-\vec{a}] = 0$

18. (c) $\vec{a} = \hat{i}-\hat{k}, \vec{b} = x\hat{i}+\hat{j}+(1-x)\hat{k}$,

$\vec{c} = y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$= 1(1+x-y-x+x^2) - 1(x^2-y) = 1$

$\therefore$ Depends neither on x nor on y.

19. (b) $\hat{a}, \hat{b}, \hat{c}$ are units vectors.

$\therefore \quad \hat{a}.\hat{a} = \hat{b}.\hat{b} = \hat{c}.\hat{c} = 1$

Now, $x = |\hat{a}-\hat{b}|^2 + |\hat{b}-\hat{c}|^2 + |\hat{c}-\hat{a}|^2$

$= \hat{a}.\hat{a}+\hat{b}.\hat{b}-2\hat{a}.\hat{b}+\hat{b}.\hat{b}+\hat{c}.\hat{c} - 2\hat{b}.\hat{c}+\hat{c}.\hat{c}+\hat{a}.\hat{a}-2\hat{c}.\hat{a}$

$= 6-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a})$ ...(1)

Also

$\Rightarrow |\hat{a}+\hat{b}+\hat{c}| \geq 0 \Rightarrow |\hat{a}+\hat{b}+\hat{c}|^2 \geq 0$

$\Rightarrow \hat{a}.\hat{a}+\hat{b}.\hat{b}+\hat{c}.\hat{c}+2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \geq 0$

$\Rightarrow 3+2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \geq 0 \Rightarrow 2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \geq -3$

$\Rightarrow -2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \leq 3$

$\Rightarrow 6-2(\hat{a}.\hat{b}+\hat{b}.\hat{c}+\hat{c}.\hat{a}) \leq 9$ ...(2)

From (1) and (2), $x \leq 9 \therefore x$ does not exceed 9

20. (b) Given that $\vec{a}$ and $\vec{b}$ are two unit vectors

$\therefore \quad |\vec{a}| = 1$ and $|\vec{b}| = 1$

Also, given that $(\vec{a}+2\vec{b}) \perp (5\vec{a}-4\vec{b})$

$\Rightarrow (\vec{a}+2\vec{b}).(5\vec{a}-4\vec{b}) = 0$

$\Rightarrow 5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a}.\vec{b} + 10\vec{b}.\vec{a} = 0$

$\Rightarrow 5-8+6\vec{a}.\vec{b} = 0 \Rightarrow 6|\vec{a}||\vec{b}|\cos\theta = 3$

[where θ is the angle between $\vec{a}$ and $\vec{b}$]

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

21. (c) Given that $\vec{v} = 2\hat{i}+\hat{j}-\hat{k}$ and $\vec{w} = \hat{i}+3\hat{k}$ and u is a unit vector $\therefore |\vec{u}| = 1$

Now, $[\vec{u}\,\vec{v}\,\vec{w}] = \vec{u}.(\vec{v}\times\vec{w})$

$= \vec{u}.(2\hat{i}+\hat{j}-\hat{k}) \times (\hat{i}+3\hat{k})$

$= \vec{u}.(3\hat{i}-7\hat{j}-\hat{k}) = \sqrt{3^2+7^2+1^2}\cos\theta$

which is max. when $\cos\theta = 1$

$\therefore$ Max. value of $[\vec{u}\,\vec{v}\,\vec{w}] = \sqrt{59}$

22. (a) As the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane $2x-4y+z = 7$, the point (4, 2, k) through which line passes must also lie on the given plane and hence $2\times4-4\times2+k = 7 \Rightarrow k = 7$

23. (c) Vol. of parallelopiped formed by

$\vec{u} = \hat{i}+a\hat{j}+\hat{k}, \vec{v} = \hat{j}+a\hat{k}, \vec{w} = a\hat{i}+\hat{k}$ is

$$V = [\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$= 1(1-0) - a(0-a^2) + 1(0-a) = 1+a^3-a$

For V to be min $\frac{dV}{da} = 0$

$\Rightarrow 3a^2-1 = 0 \Rightarrow a = \pm\frac{1}{\sqrt{3}}$

24. (c) $\because (\vec{a}\times\vec{b})\times\vec{a} = (\vec{a}.\vec{a})\vec{b} - (\vec{a}.\vec{b})\vec{a}$

$\therefore \quad (\hat{j}-\hat{k})\times(\hat{i}+\hat{j}+\hat{k}) = (\sqrt{3})^2(\vec{b}) - (\hat{i}+\hat{j}+\hat{k})$

$\Rightarrow 3\vec{b} = 3\hat{i} \Rightarrow \vec{b} = \hat{i}$

25. (b) $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

$\Rightarrow x = 2\lambda+1, y = 3\lambda-1$ and $z = 4\lambda+1$

$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

$\Rightarrow x = 3+\mu, y = k+2\mu$ and $z = \mu$

Since above lines intersect

$\Rightarrow 2\lambda+1 = 3+\mu$ ...(1)

$3\lambda-1 = 2\mu+k$ ...(2)

$\mu = 4\lambda+1$ ...(3)

Solving (1) and (3) and putting the value of λ and μ in (2) we get, $k = \frac{9}{2}$

26. (c) Any vector coplanar to $\vec{a}$ and $\vec{b}$ can be written as $\vec{r} = \vec{a}+\lambda\vec{b}$

$\vec{r} = (1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(1+\lambda)\hat{k}$

Since $\vec{r}$ is orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$

$\Rightarrow 5(1+2\lambda)+2(-1+\lambda)+6(1+\lambda) = 0$

$\Rightarrow 9+18\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$

$\therefore$ $\bar{r}$ is $3\hat{j}-\hat{k}$

Since $\hat{r}$ is a unit vector, $\therefore$ $\hat{r}=\frac{3\hat{j}-\hat{k}}{\sqrt{10}}$

27. **(d)** Let the eq^n of variable plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ which meets the axes at $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0. 0, c)$.

$\therefore$ Centroid of ΔABC is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

and it satisfies the relation

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k \Rightarrow \frac{9}{a^2}+\frac{9}{b^2}+\frac{9}{c^2}=k$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{z^2}=\frac{k}{9} \quad ...(1)$$

Also given that the distance of plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ from $(0, 0, 0)$ is 1 unit.

$$\Rightarrow \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=1 \Rightarrow \frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=1 \quad ...(2)$$

From (1) and (2), we get $\frac{k}{9}=1$ i.e. $k=9$

28. **(b)** We observe that

$$\vec{a}.\vec{b}_1=\vec{a}.\vec{b}-\left(\frac{\vec{b}.\vec{a}}{|\vec{a}|^2}\right)\vec{a}.\vec{a}=\vec{a}.\vec{b}-\vec{a}.\vec{b}=0$$

$$\vec{a}.\vec{c}_2=\vec{a}.\left(\vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}.\vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1\right)$$

$$=\vec{a}.\vec{c}-\vec{c}.\frac{\vec{a}.\vec{c}}{|\vec{a}|^2}|\vec{a}|^2-\frac{\vec{c}.\vec{b}_1}{|\vec{b}_1|^2}(\vec{a}.\vec{b}_1)$$

$$=\vec{a}.\vec{c}-\vec{a}.\vec{c}-0=0 \qquad [\because \vec{a}.\vec{b}_1=0]$$

And $\vec{b}_1.\vec{c}_2=\vec{b}_1.\left(\vec{c}-\frac{\vec{c}.\vec{a}}{|\vec{a}|^2}\vec{a}-\frac{\vec{c}.\vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1\right)$

$$=\vec{b}_1.\vec{c}-\frac{(\vec{c}.\vec{a})(\vec{b}_1.\vec{a})}{|\vec{a}|^2}-\frac{\vec{c}.\vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1.\vec{b}_1$$

$$=\vec{b}_1.\vec{c}-0-\vec{b}_1.\vec{c} \qquad (\text{Using } \vec{b}_1.\vec{a}=0)=0$$

Hence $\vec{a}.\vec{b}_1=\vec{a}.\vec{c}_2=\vec{b}_1.\vec{c}_2=0$

$\Rightarrow$ $(\vec{a},\vec{b}_1,\vec{c}_2)$ is a set of orthogonal vectors.

29. **(d)** The equation of plane through the point $(1, -2, 1)$ and perpendicular to the planes $2x-2y+z=0$ and $x-y+2z=4$ is given by

$$\begin{vmatrix} x-1 & y+2 & z-1 \\ 2 & -2 & 1 \\ 1 & -1 & 2 \end{vmatrix}=0 \Rightarrow x+y+1=0$$

It's distance from the point $(1, 2, 2)$ is

$$\left|\frac{1+2+1}{\sqrt{2}}\right|=2\sqrt{2}.$$

30. **(a)** A vector in the plane of $\vec{a}$ and $\vec{b}$ is

$$\vec{u}=\vec{a}+\lambda\vec{b}=(1+\lambda)\hat{i}+(2-\lambda)\hat{j}+(1+\lambda)\hat{k}$$

Projection of $\vec{u}$ on $\vec{c}=\frac{1}{\sqrt{3}} \Rightarrow \frac{\vec{u}.\vec{c}}{|\vec{c}|}=\frac{1}{\sqrt{3}}$

$\Rightarrow \vec{u}.\vec{c}=1 \Rightarrow |1+\lambda+2-\lambda-1-\lambda|=1$

$\Rightarrow |2-\lambda|=1 \Rightarrow \lambda=1$ or 3

$\Rightarrow \vec{u}=2\hat{i}+\hat{j}+2\hat{k}$ or $4\hat{i}-\hat{j}+4\hat{k}$

31. **(c)** We know that three vector are coplanar if their scalar triple product is zero.

$$\Rightarrow \begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix}=0$$

$R_1 \to R_1+R_2+R_3$

$$\Rightarrow \begin{vmatrix} 2-\lambda^2 & 2-\lambda^2 & 2-\lambda^2 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix}=0$$

$$\Rightarrow (2-\lambda^2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix}=0$$

$$\Rightarrow (2-\lambda^2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -(1+\lambda^2) & 0 \\ 0 & 0 & -(1+\lambda^2) \end{vmatrix}=0$$

(R_2-R_1, R_3-R_1)

$\Rightarrow (2-\lambda^2)(1+\lambda^2)^2=0 \Rightarrow \lambda=\pm\sqrt{2}$

$\therefore$ Two real solutions.

32. **(b)** Since, $\vec{a}+\vec{b}+\vec{c}=0$ and $\vec{a},\vec{b},\vec{c}$ are unit vectors, therefore $\vec{a},\vec{b},\vec{c}$ form an equilateral triangle.

$\Rightarrow \vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{0}$

$\Rightarrow \vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{0} \Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

Similarly, $\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\therefore$ $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Also since $\vec{a},\vec{b},\vec{c}$ are non parallel (these form an equilateral Δ).

$\therefore$ $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$

33. (a) We know that the volume of a parallelopipe with coterminus edges as the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ is given by

$$V=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c}\\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c}\\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c}\end{vmatrix}$$

$$\Rightarrow V^2=\begin{vmatrix}1 & 1/2 & 1/2\\ 1/2 & 1 & 1/2\\ 1/2 & 1/2 & 1\end{vmatrix}=\frac{1}{2}\Rightarrow V=\frac{1}{\sqrt{2}}$$

34. (a) Given $\overrightarrow{OP}=\hat{a}\cos t+\hat{b}\sin t$

$\Rightarrow |\overrightarrow{OP}|^2=\cos^2 t+\sin^2 t+2\hat{a}.\hat{b}\sin t\cos t$

$\Rightarrow |\overrightarrow{OP}|^2=1+\hat{a}.\hat{b}\sin 2t\le 1+\hat{a}.\hat{b}$ (Max . at $t=\frac{\pi}{4}$)

$\therefore |\overrightarrow{OP}|_{max}=\sqrt{1+\hat{a}.\hat{b}}$

Also $\hat{u}=|\widehat{OP}|_{max}$

Maximum occurs at $t=\frac{\pi}{4}$

$\therefore |\overrightarrow{OP}|_{max}=\frac{\hat{a}+\hat{b}}{\sqrt{2}}$ $\therefore |\hat{O}P|_{max}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$

Hence $\hat{u}=\frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M=\sqrt{1+\hat{a}.\hat{b}}$

35. (a) Given that $P(3,2,6)$ is a point in space and Q is a point on line

$\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$

or $\frac{x-1}{-3}=\frac{y+1}{1}=\frac{z-2}{5}=\mu$

Let coordinates of Q be $(-3\mu+1,\mu-1,5\mu+2)$

$\therefore$ d.r's of $\overrightarrow{PQ}=-3\mu-2,\ \mu-3,\ 5\mu-4$

As $\overrightarrow{PQ}$ is parallel to the plane $x-4y+3z=1$

$\therefore$ $1.(-3\mu-2)-4.(\mu-3)+3.(5\mu-4)=0$

$\Rightarrow$ $8\mu=2$ or $\mu=\frac{1}{4}$

36. (c) $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are unit vectors,

Let $\vec{a}\times\vec{b}=(\sin\alpha)\overrightarrow{n_1}$ and $\vec{c}\times\vec{d}=(\sin\beta)\overrightarrow{n_2}$

then $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=1$

$\Rightarrow$ $(\sin\alpha)\overrightarrow{n_1}.(\sin\beta)\overrightarrow{n_2}=1$

$\Rightarrow$ $\sin\alpha\sin\beta\ \overrightarrow{n_1}.\overrightarrow{n_2}=1\Rightarrow\sin\alpha\sin\beta\cos\gamma=1$

where γ is the angle between $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$.

$\alpha=\frac{\pi}{2},\ \beta=\frac{\pi}{2}$ and $\gamma=0°$

Now $\gamma=0°\Rightarrow\vec{a}\times\vec{b}\parallel\vec{c}\times\vec{d}$

Let $\vec{a}\times\vec{b}=\lambda(\vec{c}\times\vec{d})\Rightarrow(\vec{a}\times\vec{b}).\vec{c}=\lambda(\vec{c}\times\vec{d}).\vec{c}=0$

and $(\vec{a}\times\vec{b}).\vec{d}=\lambda(\vec{c}\times\vec{d}).\vec{d}=0$

$\therefore$ $\vec{a},\vec{b},\vec{c}$ are coplanar and $\vec{a},\vec{b},\vec{d}$ are coplanar

$\Rightarrow$ $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar

Also $\alpha=90°\Rightarrow\vec{a}\perp\vec{b}$ and $\beta=90°\Rightarrow\vec{c}\perp\vec{d}$

But angle between $\vec{a}$ and $\vec{c}$ is $\pi/3$ ($\because\ \vec{a}.\vec{c}=\frac{1}{2}$)

So, angle between $\vec{b}$ and $\vec{d}$ should also be $\pi/3$. Hence $\vec{b}$ and $\vec{d}$ are non parallel.

37. (c) The line has +ve and equal direction cosines, these are $\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ or direction ratios are 1, 1, 1. Also the lines passes through $P(2,-1,2)$.

$\therefore$ Equation of line is

$$\frac{x-2}{1}=\frac{y+1}{1}=\frac{z-2}{1}=\lambda\ \text{(say)}$$

Let $Q(\lambda+2,\lambda-1,\lambda+2)$ be a point on this line where it meets the plane $2x+y+z=9$

Then Q must satisfy the eq^n of plane

i.e. $2(\lambda+2)+\lambda-1+\lambda+2=9\Rightarrow\lambda=1$

$\therefore$ Q has coordintes $(3,0,3)$

Hence the length of line segments PQ

$=\sqrt{(2-3)^2+(-1-0)^2+(2-3)^2}=\sqrt{3}$

38. (a) We have $\overline{PQ}=6\hat{i}+\hat{j},\ \overline{QR}=-\hat{i}+3\hat{j},\ \overline{SR}=6\hat{i}+\hat{j},$

$\overline{PS}=-\hat{i}+3\hat{j}\Rightarrow\overline{PQ}=\overline{SR};\ \overline{QR}=\overline{PS}$ and $\overline{PQ}.\overline{PS}\neq 0$

Also $|\overline{PQ}|\neq|\overline{QR}|$

$\Rightarrow$ PQRS is a parallelogram but neither a rhombus nor a rectangle.

39. (c) Plane containing two lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is given by

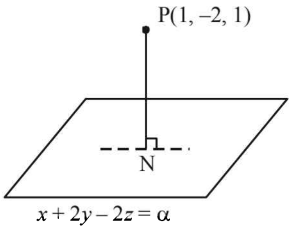
$$\begin{vmatrix}x & y & z\\ 3 & 4 & 2\\ 4 & 2 & 3\end{vmatrix}=0\Rightarrow 8x-y-10z=0$$

Now equation of plane containing the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane $8x-y-10z=0$ is

$$\begin{vmatrix}x & y & z\\ 2 & 3 & 4\\ 8 & -1 & -10\end{vmatrix}=0$$

$\Rightarrow -26x+52y-26z=0$ or $x-2y+z=0$

40. (a) As perpendicular distance of $x+2y-2z=\alpha$ from the point $(1,-2,1)$ is 5



$\therefore \left|\dfrac{1-4-2-\alpha}{3}\right| = 5$

$\Rightarrow \dfrac{-5-\alpha}{3} = 5 \text{ or } -5$

$\Rightarrow \alpha = -20 \text{ or } 10$

As $\alpha > 0 \Rightarrow \alpha = 10$

$\therefore$ Plane becomes $x+2y-2z-10=0$

Equation of PN is $\dfrac{x-1}{1} = \dfrac{y+2}{2} = \dfrac{z-1}{-2} = \lambda$

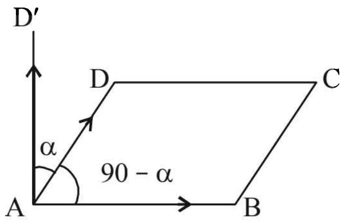
For some value of λ, $N(\lambda+1, 2\lambda-2, -2\lambda+1)$

It lies on $x+2y-2z-10=0$

$\therefore \lambda+1+4\lambda-4+4\lambda-2=10 \Rightarrow 9\lambda=15 \Rightarrow \lambda=5/3$

$\therefore N\left(\dfrac{2}{3}, \dfrac{4}{3}, \dfrac{-7}{3}\right)$

41. (b)



$$\sin(90-\alpha) = \frac{\left|\overrightarrow{AB}\times\overrightarrow{AD}\right|}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AD}\right|}$$

Where, $\overrightarrow{AB}\times\overrightarrow{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 10 & 11 \\ -1 & 2 & 2 \end{vmatrix} = -2\hat{i}-15\hat{j}+14\hat{k}$

$\therefore \left|\overrightarrow{AB}\times\overrightarrow{AD}\right| = \sqrt{4+225+196} = \sqrt{425}$

$\left|\overrightarrow{AB}\right| = \sqrt{4+100+121} = \sqrt{225} = 15$

$\left|\overrightarrow{AD}\right| = \sqrt{1+4+4} = 3$

$\therefore \sin(90-\alpha) = \dfrac{\sqrt{425}}{15\times 3} = \dfrac{\sqrt{17}}{9} \Rightarrow \cos\alpha = \dfrac{\sqrt{17}}{9}$

42. (c) As $\vec{v}$ lies in the plane of $\vec{a}$ and $\vec{b}$

$\therefore \vec{v} = \lambda\vec{a}+\mu\vec{b}$

$\Rightarrow \vec{v} = (\lambda+\mu)\,\hat{i}+(\lambda-\mu)\hat{j}+(\lambda+\mu)\hat{k}$

$\because$ Projection of $\vec{v}$ on $\vec{c}$ is $\dfrac{1}{\sqrt{3}}$

$\therefore \dfrac{\vec{v}.\vec{c}}{|\vec{c}|} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{(\lambda+\mu)-(\lambda-\mu)-(\lambda+\mu)}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \mu-\lambda = 1 \text{ or } \mu = \lambda+1$

$\Rightarrow \vec{v} = (2\lambda+1)\hat{i}-\hat{j}+(2\lambda+1)\hat{k}$

For $\lambda=1$, $\vec{v} = 3\hat{i}-\hat{j}+3\hat{k}$.

43. (a) Equation of st. line joining Q (2, 3, 5) and $R(1,-1,4)$ is

$\dfrac{x-2}{-1} = \dfrac{y-3}{-4} = \dfrac{z-5}{1} = \lambda$

Let $P(-\lambda+2, -4\lambda+3, -\lambda+5)$

As P lies on $5x-4y-z=1$

$\therefore \quad -5\lambda+10+16\lambda-12+\lambda-5=1$

$\Rightarrow \quad 12\lambda = 8 \Rightarrow \lambda = \dfrac{2}{3} \qquad \therefore P = \left(\dfrac{4}{3}, \dfrac{1}{3}, \dfrac{13}{3}\right)$

Now let point S on QR be

$(-\mu+2, -4\mu+3, -\mu+5)$

$\because$ S is the foot of perpendicular drawn from $T(2,1,4)$ to QR, where dr's of ST are $\mu, 4\mu-2, \mu-1$ and dr's of QR are $-1,-4,-1$

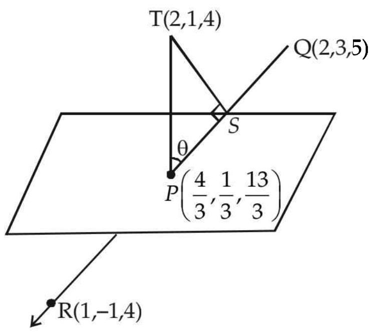
$\therefore \quad -\mu-16\mu+8-\mu+1=0 \Rightarrow 18\mu=9 \Rightarrow \mu=\dfrac{1}{2}$

$\therefore \quad S = \left(\dfrac{3}{2}, 1, \dfrac{9}{2}\right)$

$\therefore$ Distance between P and S

$= \sqrt{\left(\dfrac{4}{3}-\dfrac{3}{2}\right)^2+\left(\dfrac{1}{3}-1\right)^2+\left(\dfrac{13}{3}-\dfrac{9}{2}\right)^2}$

$= \sqrt{\dfrac{1}{36}+\dfrac{4}{9}+\dfrac{1}{36}} = \dfrac{1}{\sqrt{2}}$



44. (a) The plane passing through the intersection line of given planes is

$(x+2y+3z-2)+\lambda(x-y+z-3)=0$

or $\quad (1+\lambda)x+(2-\lambda)y+(3+\lambda)z+(-2-3\lambda)=0$

Its distance from the point $(3,1,-1)$ is $\dfrac{2}{\sqrt{3}}$

$\therefore \left|\dfrac{3(1+\lambda)+1(2-\lambda)-1(3+\lambda)+(-2-3\lambda)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right| = \dfrac{2}{\sqrt{3}}$

$$\Rightarrow \left|\frac{-2\lambda}{\sqrt{3\lambda^2+4\lambda+14}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow 3\lambda^2+4\lambda+14 = 3\lambda^2 \Rightarrow \lambda = -\frac{7}{2}$$

$\therefore$ Required equation of plane is

$$(x+2y+3z-2) - \frac{7}{2}(x-y+z-3) = 0$$

or $5x - 11y + z = 17$

45. (c) Given that $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$

$\Rightarrow (\vec{a}+\vec{b})\times(2\hat{i}+3\hat{j}+4\hat{k}) = \vec{0}$

But neither $\vec{a}+\vec{b}$ nor $2\hat{i}+3\hat{j}+4\hat{k}$ is a null vector

$\therefore (\vec{a}+\vec{b}) \,||\, (2\hat{i}+3\hat{j}+4\hat{k}) \Rightarrow \vec{a}+\vec{b} = \lambda(2\hat{i}+3\hat{j}+4\hat{k})$

Also given $|\vec{a}+\vec{b}| = \sqrt{29} \Rightarrow \lambda = \pm 1$

$\therefore \vec{a}+\vec{b} = \pm(2\hat{i}+3\hat{j}+4\hat{k})$

$\therefore (\vec{a}+\vec{b}).(-7\hat{i}+2\hat{i}+3\hat{k}) = \pm 4$

46. (c) P, the image of point (3, 1, 7) in the plane $x-y+z=3$ is given by

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = \frac{-2(3-1+7-3)}{1^2+1^2+1^2}$$

$$\Rightarrow \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = -4$$

$\Rightarrow$ $x=-1, y=5, z=3$

$\therefore$ P(–1, 5, 3)

Now equation of plane through (–1, 5, 3) and containing the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

$$\begin{vmatrix} x & y & z \\ -1 & 5 & 3 \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow -x+4y-7z = 0$$

or $x - 4y + 7z = 0$

D. MCQs with ONE or MORE THAN ONE Correct

1. (c) We are given that, $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ $\quad \vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

Then $$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = [\vec{a}\vec{b}\vec{c}]^2 = (\vec{a}\times\vec{b}.\vec{c})^2$$

$$= (|\vec{a}\times\vec{b}|.1\cos 0°)^2 \quad \left[\begin{array}{l}\vec{c} \text{ is a unit vector, } \therefore |\vec{c}|=1 \\ \text{Also } \vec{c} \text{ is} \perp \text{to } \vec{a} \text{ as well} \\ \text{as to } \vec{b}, \therefore \vec{c}\perp(\vec{a}\times\vec{b})\end{array}\right.$$

$$= (|\vec{a}\times\vec{b}|)^2 = \left(|\vec{a}||\vec{b}|.\sin\frac{\pi}{6}\right)^2$$

[$\because$ angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$]

$$= \left(\frac{1}{2}\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}\right)^2$$

$$= \frac{1}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$$

2. (b) We know that if $\hat{n}$ is $\perp$ to $\vec{a}$ as well as $\vec{b}$ then

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} \text{ or } \frac{\vec{b}\times\vec{a}}{|\vec{b}\times\vec{a}|}$$

as $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ represent two vectors in opp. directions.

$\therefore$ We have two possible values of $\hat{n}$

3. (a, c) We have

$\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \vec{b} = \hat{i}+2\hat{j}-\hat{k}, \hat{c} = \hat{i}+\hat{j}-2\hat{k}$

Any vector in the plane of $\vec{b}$ and $\vec{c}$ is $\vec{u} = \vec{b}+\lambda\vec{c}$

$= (\hat{i}+2\hat{j}-\hat{k}) + \lambda(\hat{i}+\hat{j}-2\hat{k})$

$= (1+\lambda)\hat{i} + (2+\lambda)\hat{j} - (1+2\lambda)\hat{k}$

Given magnitude of projection of $\vec{u}$ on $\vec{a}$ is $\sqrt{\frac{2}{3}}$

$$\Rightarrow \sqrt{\frac{2}{3}} = \left|\frac{\vec{u}.\vec{a}}{|\vec{a}|}\right| \Rightarrow \sqrt{\frac{2}{3}} = \left|\frac{2(1+\lambda)-(2+\lambda)-(1+2\lambda)}{\sqrt{6}}\right|$$

$\Rightarrow |-\lambda-1| = 2 \Rightarrow \lambda+1 = 2$ or $\lambda+1 = -2$

$\Rightarrow$ $\lambda = 1$ or $\lambda = -3$

$\therefore$ The required vector is either,

$2\hat{i}+3\hat{j}-3\hat{k}$ or $-2\hat{i}-\hat{j}+5\hat{k}$

4. (a, c, d) $|\vec{a}|^2 = \frac{1}{9}(4+4+1) = 1 \Rightarrow |\vec{a}| = 1$

Let $\vec{b} = 2\hat{i}-4\hat{j}+\hat{3}$ then

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{5}{\sqrt{29}} \Rightarrow \theta \neq \frac{\pi}{3}$$

Let $\vec{c} = -\hat{i}+\hat{j}-\frac{1}{2}\hat{k} = \frac{-3}{2}\hat{a} \Rightarrow \vec{c} \,||\, \vec{a}$

Let $\vec{d} = 3\hat{i}+2\hat{j}+2\hat{k}$ then $\vec{a}.\vec{d} = 0 \Rightarrow \vec{a}\perp\vec{d}$

5. (d) Given that, $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = 4\hat{i}+3\hat{j}+4\hat{k}$

and $\vec{c} = \hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent,

NOTE THIS STEP:

$\therefore$ $\vec{c} = l\vec{a}+m\vec{b}$ for some scalars l and m not all zeros.

$\hat{i}+\alpha\hat{j}+\beta\hat{k} = (l+4m)\hat{i} + (l+3m)\hat{j} + (l+4m)\hat{k}$

$\Rightarrow$ $l+4m = 1$ ...(1)

$l+3m = \alpha$ ...(2)

$l+4m = \beta$ ...(3)

From (1) and (3) we have, $\beta = 1$

Also given that $|\vec{c}| = \sqrt{3} \Rightarrow 1+\alpha^2+\beta^2 = 3$

Substituting the value of β we get $\alpha^2 = 1$

$\Rightarrow \quad \alpha = \pm 1$

6. **(c)** $[\vec{u}\,\vec{v}\,\vec{w}] = [v\,w\,u] = [\vec{w}\,\vec{u}\,\vec{v}]$

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but $[\vec{v}\,\vec{u}\,\vec{w}] = -[\vec{u}\,\vec{v}\,\vec{w}]$

III

7. **(a, c)** As dot product of two vectors gives a scalar quantity.

8. **(a, c)** We have

$\vec{v} = \vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n} = \sin\theta\hat{n}$

[$\because \vec{a}$ and $\vec{b}$ are unit vectors.]

$\therefore \quad |\vec{v}| = \sin\theta$

Now, $\vec{u} = \vec{a} - (\vec{a}.\vec{b})\vec{b}$

$= \vec{a} - \vec{b}\cos\theta \qquad$ (where $\vec{a}.\vec{b} = \cos\theta$)

$\therefore \quad |\vec{u}|^2 = |\vec{a} - \vec{b}\cos\theta|^2 = 1 + \cos^2\theta - 2\cos\theta.\cos\theta$

$= 1 - \cos^2\theta = \sin^2\theta = |v|^2 \Rightarrow |\vec{u}| = |\vec{v}|$

Also, $\vec{u}.\vec{b} = \vec{a}.\vec{b} - (\vec{a}.\vec{b})\,(\vec{b}.\vec{b}) = \vec{a}.\vec{b} - \vec{a}.\vec{b} = 0 \therefore\ |\vec{u}.\vec{b}| = 0$

$\therefore \quad |\vec{v}| = |\vec{u}| + |\vec{u}.\vec{b}|$ is also correct

9. **(b,d)** Normal to plane P_1 is

$\vec{n}_1 = (2\hat{i} + 3\hat{k})\times(4\hat{j} - 3\hat{k}) = -18\hat{i}$

Normal to plane P_2 is

$\vec{n}_2 = (\hat{j} - \hat{k})\times(3\hat{i} + 3\hat{j}) = 3\hat{i} - 3\hat{j} - 3\hat{k}$

$\therefore \quad \vec{A}$ is parallel to $\pm(\hat{n}_1 \times \hat{n}_2) = \pm(-54\hat{j} + 54\hat{k})$

Now, angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is given by

$$\cos\theta = \pm\frac{(-54\hat{j} + 54\hat{k}).(2\hat{i} + \hat{j} - 2\hat{k})}{54\sqrt{2}.3} = \pm\frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

10. **(a,d)** Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k},\ \vec{b} = \hat{i} + 2\hat{j} + \hat{k},\ \vec{c} = \hat{i} + \hat{j} + \hat{k}$

$\because$ Required vector is coplanar with $\vec{a}$ and $\vec{b}$

$\therefore \vec{r} = \lambda\vec{a} + \mu\vec{b}$

or $\vec{r} = (\lambda + \mu)\,\hat{i} + (\lambda + 2\mu)\,\hat{j} + (2\lambda + \mu)\,\hat{k}$

As $\vec{r} \perp \vec{c} \Rightarrow \vec{r}.\vec{c} = 0$

$\Rightarrow \lambda + \mu + \lambda + 2\mu + 2\lambda + \mu = 0 \Rightarrow \lambda + \mu = 0 \Rightarrow \lambda = -\mu$

$\therefore \vec{r} = \mu(\hat{j} - \hat{k})$

For $\mu = 1$, we get $\vec{r} = \hat{j} - \hat{k}$

and for $\mu = -1$, we get $\vec{r} = -\hat{j} + \hat{k}$

$\therefore$ a and d are the correct options.

11. **(b, c)** For given lines to be coplanar, we should have

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \Rightarrow k = \pm 2$$

For $k = 2$, obviously the plane $y + 1 = z$ is common in both lines.

For $k = -2$, the plane is given by

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0 \Rightarrow y + z + 1 = 0$$

12. **(b, d)** The given lines are

$$\ell_1 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} = t$$

$$\ell_2 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = s$$

Let direction ratios of ℓ be a, b, c then as $\ell \perp \ell_1$ and ℓ_2

$\therefore \quad a + 2b + 2c = 0$

$2a + 2b + c = 0$

$$\Rightarrow \frac{a}{-2} = \frac{b}{3} = \frac{c}{-2}$$

$$\therefore \quad \ell : \frac{x}{2} = \frac{y}{-3} = \frac{z}{2} = \lambda$$

Any point on ℓ_1 is $(t + 3, 2t - 1, 2t + 4)$ and any point on ℓ is $(2\lambda, -3\lambda, 2\lambda)$

$\therefore$ For Intersection point P of ℓ and ℓ_1

$t + 3 = 2\lambda,\ 2t - 1 = -3\lambda,\ 2t + 4 = 2\lambda$

$\Rightarrow t = -1,\ \lambda = 1 \qquad \therefore P(2, -3, 2)$

Any point Q on ℓ_2 is $(2s + 3, 2s + 3, s + 2)$

As per question $PQ = \sqrt{17}$

$\Rightarrow (2s + 1)^2 + (2s + 6)^2 + s^2 = 17$

$$\Rightarrow 9s^2 + 28s + 20 = 0 \Rightarrow s = -2, \frac{-10}{9}$$

$$\therefore \text{ Point } Q \text{ can be } (-1, -1, 0) \text{ and } \left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$$

13. **(a, d)** $$L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

As L_1, L_2 are coplanar, therefore

$$\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$$

$\Rightarrow (5 - \alpha)[6 - 5\alpha + \alpha^2 - 2] = 0$

$\Rightarrow (5 - \alpha)(\alpha - 1)(\alpha - 4) = 0 \Rightarrow \alpha = 1, 4, 5.$

14. **(a, b, c)** $\left|\vec{x}\right| = \left|\vec{y}\right| = \left|\vec{z}\right| = \sqrt{2}$

Angle between each pair is $\frac{\pi}{3}$

$$\vec{a} = \lambda\left[\vec{x} \times \left(\vec{y} \times \vec{z}\right)\right]$$

$= \lambda\left[\left(\vec{x}\cdot\vec{z}\right)\vec{y} - \left(\vec{x}\cdot\vec{y}\right)\vec{z}\right]$

$= \lambda\left[\left(\sqrt{2}.\sqrt{2}\cos\frac{\pi}{3}\right)\vec{y} - \left(\sqrt{2}.\sqrt{2}\cos\frac{\pi}{3}\right)\vec{z}\right]$

$= \lambda\left(\vec{y} - \vec{z}\right)$

$\vec{b} = \mu\left[\vec{y}\times\left(\vec{z}\times\vec{x}\right)\right]$

$= \mu\left[\left(\vec{y}.\vec{x}\right)\vec{z} - \left(\vec{y}.\vec{z}\right)\vec{x}\right]$

$= \mu\left[\left(\sqrt{2}.\sqrt{2}.\cos\frac{\pi}{3}\right)\vec{z} - \left(\sqrt{2}.\sqrt{2}.\cos\frac{\pi}{3}\right)\vec{x}\right]$

$= \mu\left(\vec{z} - \vec{x}\right)$

Now $\vec{b}.\vec{z} = \mu\left[\vec{z}.\vec{z} - \vec{x}.\vec{z}\right] = \mu(2-1) = \mu$

$\therefore \vec{b} = \left(\vec{b}.\vec{z}\right)\left(\vec{z} - \vec{x}\right)$ is correct

Also $\vec{a}.\vec{y} = \lambda\left(\vec{y}.\vec{y} - \vec{z}.\vec{y}\right) = \lambda(2-1) = \lambda$

$\therefore \vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{y} - \vec{z}\right)$ is also correct

$\vec{a}.\vec{b} = \lambda\mu\left(\vec{y}.\vec{z} - \vec{y}.\vec{x} - \vec{z}.\vec{z} + \vec{z}.\vec{x}\right)$

$= \lambda\mu(1-1-2+1) = -\lambda\mu = -\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$

$\therefore$ (c) is correct.

$-\left(\vec{a}.\vec{y}\right)\left(\vec{z} - \vec{y}\right) = \lambda\left(\vec{z} - \vec{y}\right) = -\vec{a}$

(d) is not correct.

15. (c) Lines are $x = y, z = 1$

or $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{0} = \alpha$ ...(1)

and $y = -x, z = -1$

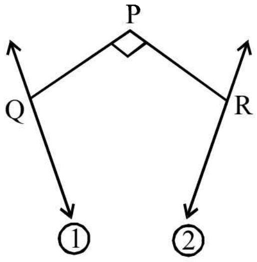
or $\frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = \beta$ ...(2)

Let $Q(\alpha, \alpha, 1)$ and $R(-\beta, \beta, -1)$

Direction ratios of PQ are $\lambda - \alpha, \lambda - \alpha, \lambda - 1$

and direction ratios of PR are $\lambda + \beta, \lambda - \beta, \lambda + 1$

$\because PQ$ is perpendicular to line (1)



$\therefore -(\lambda+\beta) + \lambda - \beta = 0 \Rightarrow \beta = 0$

$\therefore$ dr's of PQ are $0, 0, \lambda - 1$

and dr's of PR are $\lambda, \lambda, \lambda + 1$

$\because \angle QPR = 90° \Rightarrow (\lambda - 1)(\lambda + 1) = 0 \Rightarrow \lambda = 1$ or -1

But for $\lambda = 1$, we get point Q itself

$\therefore$ we take $\lambda = -1$

16. (b, d) $P_3 : x + \lambda y + z - 1 = 0$

Also $\left|\frac{\lambda - 1}{\sqrt{2+\lambda^2}}\right| = 1 \Rightarrow \lambda^2 - 2\lambda + 1 = \lambda^2 + 2 \Rightarrow \lambda = \frac{-1}{2}$

And $\left|\frac{\alpha + \lambda\beta + \gamma - 1}{\sqrt{2+\lambda^2}}\right| = 2 \Rightarrow \frac{\alpha - \frac{1}{2}\beta + \gamma - 1}{\frac{3}{2}} = \pm 2$

$\Rightarrow \alpha - \frac{1}{2}\beta + \gamma - 1 = \pm 3 \Rightarrow 2\alpha - \beta + 2\gamma - 2 = \pm 6$

$\Rightarrow 2\alpha - \beta + 2\gamma - 8 = 0$ or $2\alpha - \beta + 2\gamma + 4 = 0$

17. (a, b) $L : \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = \lambda$

Where $\left.\begin{aligned} a + 2b - c = 0 \\ 2a - b + c = 0 \end{aligned}\right\}$ *As L is parallel to both P_1 and P_2.*

$\Rightarrow \frac{a}{1} = \frac{b}{-3} = \frac{c}{-5}$

$\therefore$ Any point on line L is $(\lambda, -3\lambda, -5\lambda)$

Equation of line perpendicular to P_1 drawn from any point on L is

$\frac{x-\lambda}{1} = \frac{y+3\lambda}{2} = \frac{z+5\lambda}{-1} = \mu$

$\therefore M(\mu + \lambda, 2\mu - 3\lambda, -\mu - 5\lambda)$

But M lies on P_1,

$\therefore \mu + \lambda + 4\mu - 6\lambda + \mu + 5\lambda + 1 = 0 \Rightarrow \mu = \frac{-1}{6}$

$\therefore M\left(\lambda - \frac{1}{6}, -3\lambda \frac{-1}{3}, -5\lambda + \frac{1}{6}\right)$

For locus of M,

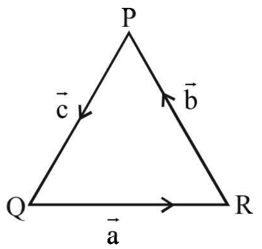
$x = \lambda - \frac{1}{6}, y = -3\lambda - \frac{1}{3}, z = 5\lambda + \frac{1}{6}$

$\Rightarrow \frac{x + 1/6}{1} = \frac{y + 1/3}{-3} = \frac{z - 1/6}{-5} = \lambda$

On checking the given point, we find $\left(0, \frac{-5}{6}, \frac{-2}{3}\right)$ and $\left(\frac{-1}{6}, \frac{-1}{3}, \frac{1}{6}\right)$ satisfy the above eqn.

18. (a, c, d)

$\vec{a}+\vec{b}+\vec{c}=\vec{o}$



$\Rightarrow \quad |\vec{b}+\vec{c}|^2 = |-\vec{a}|^2 \Rightarrow |\vec{b}|^2+|\vec{c}|^2+2\vec{b}.\vec{c}=|\vec{a}|^2$

$\Rightarrow \quad 48+|\vec{c}|^2+48=144 \Rightarrow |\vec{c}|^2=48 \Rightarrow |\vec{c}| = 4\sqrt{3}$

$\therefore \quad \frac{|\vec{c}|^2}{2}-|\vec{a}|=\frac{48}{2}-12=12$

$\frac{|\vec{c}|^2}{2}+|\vec{a}|=24\neq 30$

Also $|\vec{b}|=|\vec{c}| \Rightarrow \angle Q=\angle R$

and $\cos(180-P)=\frac{\vec{b}.\vec{c}}{|\vec{b}||\vec{c}|}=\frac{1}{2}$

$\Rightarrow \quad \angle P=120° \therefore \angle Q=\angle R=30°$

Again $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{0} \Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

$\therefore \quad |\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=2|\vec{a}\times\vec{b}|=2\times 12\times 4\sqrt{3}\times\sin 150=48\sqrt{3}$

And $\vec{a}.\vec{b}=12\times 4\sqrt{3}\times\cos 150=-72$

19. (b, c, d) The coordinates of vertices of pyramid OPQRS will be

O(0, 0, 0), P(3, 0, 0), Q(3, 3, 0), R(0, 3, 0), $S\left(\frac{3}{2},\frac{3}{2},3\right)$

dr's of OQ = 1, 1, 0

dr's of OS = 1, 1, 2

$\therefore$ acute angle between OQ and OS

$=\cos^{-1}\left(\frac{2}{\sqrt{2}\times\sqrt{6}}\right)=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\neq\frac{\pi}{3}$

Eqn of plane OQS = $\begin{vmatrix} x & y & z \\ 1 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix}=0$

$\Rightarrow 2x-2y=0$ or $x-y=0$

length of perpendicular from P (3, 0, 0) to plane $x-y=0$

is $=\left|\frac{3-0}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$

Eqn of RS : $\frac{x}{\frac{3}{2}}=\frac{y-3}{\frac{-3}{2}}=\frac{z}{3}$ or $\frac{x}{1}=\frac{y-3}{-1}=\frac{z}{2}=\lambda$

If ON is perpendicular to RS, then N $(\lambda, -\lambda+3, 2\lambda)$

$\because$ ON $\perp$ RS $\Rightarrow 1\times\lambda-1(-\lambda+3)+2\times 2\lambda=0$

$\Rightarrow \lambda=\frac{1}{2} \Rightarrow N\left(\frac{1}{2},\frac{5}{2},1\right)$

$\therefore ON=\sqrt{\frac{1}{4}+\frac{25}{4}+1}=\sqrt{\frac{15}{2}}$

20. (b,c) $|\hat{u}\times\vec{v}|=1 \Rightarrow |\vec{v}|\sin\theta=1$ ...(i)

$\hat{w}.(\hat{u}\times\vec{v})=1 \Rightarrow |\vec{v}|\sin\theta\cos\alpha=1$ ...(ii)

where α is the angle between $\hat{w}$ and a vector $\perp$ lat to $\vec{u}$ & $\vec{v}$.
From (i) and (ii) $\cos\alpha=1 \Rightarrow \alpha=0°$

$\Rightarrow$ $\hat{w}$ is perpendicular to the plane containing $\vec{u}$ & $\vec{v}$

$\Rightarrow$ $\hat{w}$ is perpendicular to $\vec{u}$

Clearly there can be infinite many choices for $\vec{v}$.

Also if $\hat{u}$ lies in xy plane i.e., $\hat{u}=u_1\hat{i}+u_2\hat{j}$ then $\hat{w}.\vec{u}=0$

$\Rightarrow u_1+u_2=0 \Rightarrow |u_1|=|u_2|$

Also if $\hat{u}$ lies in xz plane, i.e., $\hat{u}=u_1\hat{i}+u_3\hat{k}$ then $\hat{w}.\vec{u}=0$

$\Rightarrow u_1+2u_3=0 \Rightarrow |u_1|=2|u_3|$

Hence (b) and (c) are the correct options.

E. SUBJECTIVE PROBLEMS

1. Let with respect to *O*, position vectors of points *A, B, C, D, E, F* be $\vec{a},\vec{b},\vec{c},\vec{d},\vec{e},\vec{f}$.

Let perpendiculars from *A* to *EF* and from *B* to *DF* meet each other at *H*. Let position vector of H be $\vec{r}$. we join *CH*. In order to prove the statement given in question, it is sufficient to prove that *CH* is perpendicular to *DE*.

Now, as $OD\perp BC \Rightarrow \vec{d}.(\vec{b}-\vec{c})=0$

$\Rightarrow \vec{d}.\vec{b}=\vec{d}.\vec{c}$ ...(1)

As $OE\perp AC \Rightarrow \vec{e}.(\vec{c}-\vec{a})=0 \Rightarrow \vec{e}.\vec{c}=\vec{e}.\vec{a}$ ...(2)

As $OF\perp AB \Rightarrow \vec{f}.(\vec{a}-\vec{b})=0 \Rightarrow \vec{f}.\vec{a}=\vec{f}.\vec{b}$ ...(3)

Also $AH\perp EF \Rightarrow (\vec{r}-\vec{a}).(\vec{e}-\vec{f})=0$

$\Rightarrow \vec{r}.\vec{e}-\vec{r}.\vec{f}-\vec{a}.\vec{e}+\vec{a}.\vec{f}=0$ ...(4)

and $BH\perp FD \Rightarrow (\vec{r}-\vec{b}).(\vec{f}-\vec{d})=0$

$\Rightarrow \vec{r}.\vec{f}-\vec{r}.\vec{d}-\vec{b}.\vec{f}+\vec{b}.\vec{d}=0$ ...(5)

Adding (4) and (5), we get

$\vec{r}.\vec{e}-\vec{a}.\vec{e}+\vec{a}.\vec{f}-\vec{r}.\vec{d}-\vec{b}.\vec{f}+\vec{b}.\vec{d}=0$

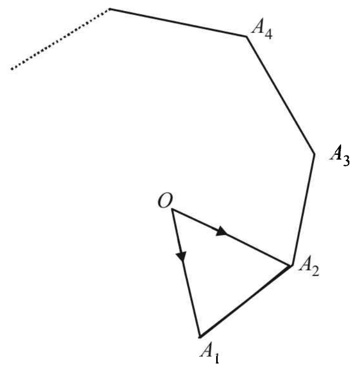
$\Rightarrow \vec{r}.(\vec{e}-\vec{d})-\vec{e}.\vec{c}+\vec{d}.\vec{c}=0$

(using (1), (2) and (3))

$\Rightarrow \vec{r}.(\vec{e}-\vec{d})-\vec{c}.(\vec{e}-\vec{d})=0 \Rightarrow (\vec{r}-\vec{c}).(\vec{e}-\vec{d})=0$

$\Rightarrow \overrightarrow{CH}.\overrightarrow{ED}=0 \Rightarrow CH\perp ED$ Hence Proved.

2. $\overrightarrow{OA_1}, \overrightarrow{OA_2}, ..., \overrightarrow{OA_n}$ all vectors are of same magnitude, say 'a' and angle between any two consecutive vector is same that is $\frac{2\pi}{n}$ radians. Let $\hat{p}$ be the unit vectors $\perp$ to the plane of the polygon.



$\therefore \quad \overrightarrow{OA_1} \times \overrightarrow{OA_2} = a^2 \sin\frac{2\pi}{n}\hat{p}$ ...(i)

Now, $\sum_{i=1}^{n-1} \overrightarrow{OA_i} \times \overrightarrow{OA_{i+1}} = \sum_{i=1}^{n-1} a^2 \sin\frac{2\pi}{n}\hat{p}$

$= (n-1)a^2 \sin\frac{2\pi}{n}\hat{p} \quad = -(n-1)[\overrightarrow{OA_2} \times \overrightarrow{OA_1}]$

[using eq^n. (i)]

$= (1-n)[\overrightarrow{OA_2} \times \overrightarrow{OA_1}] = R.H.S$

3. $(\hat{i}+\hat{j}+3\hat{k})x + (3\hat{i}-3\hat{j}+\hat{k})y + (-4\hat{i}+5\hat{j})z$

$= \lambda(x\hat{i}+y\hat{j}+z\hat{k})$

$\Rightarrow \quad x+3y-4z = \lambda x \Rightarrow (1-\lambda)x+3y-4z = 0$

$\Rightarrow \quad x-3y+5z = \lambda y \Rightarrow x-(3+\lambda)y+5z = 0$

$\Rightarrow \quad 3x+y+0z = \lambda z \Rightarrow 3x+y-\lambda z = 0$

All the above three equations are satisfied for x, y, z not all zero if

$$\begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)[3\lambda+\lambda^2-5]-3[-\lambda-15]-4[1+9+3\lambda] = 0$

$\Rightarrow \quad \lambda^3+2\lambda^2+\lambda = 0 \Rightarrow \lambda(\lambda+1)^2 = 0 \Rightarrow \lambda = 0, -1.$

4. Since vector $\vec{A}$ has components A_1, A_2, A_3, in the coordinate system $OXYZ$,

$\therefore \quad \vec{A} = \hat{i}A_1 + \hat{j}A_2 + \hat{k}A_3$

When given system is rotated through $\frac{\pi}{2}$. the new x-axis is along old y-axis and new y-axis is along the old negative x-axis z remains same as before.

Hence the components of A in the new system are $A_2, -A_1, A_3$.

$\therefore \vec{A}$ becomes $A_2\hat{i} - A_1\hat{j} + A_3\hat{k}$.

5. Then $\overrightarrow{AB} = -\hat{i}-5\hat{j}-3\hat{k}$, $\overrightarrow{AC} = -4\hat{i}+3\hat{j}+3\hat{k}$

$\overrightarrow{AD} = \hat{i}+7\hat{j}+(1-\lambda)\hat{k}$

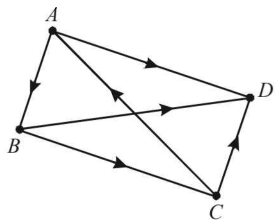
We know that A, B, C, D lie in a plane if $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar i.e. $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$

$$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad -1(3-3\lambda-21)-5(-4+4\lambda-3)-3(-28-3) = 0$

$\Rightarrow \quad 3\lambda+18-20\lambda+35+93 = 0 \Rightarrow 17\lambda = 146 \Rightarrow \lambda = \frac{146}{17}$

6. Let the position vectors of points A, B, C, D be a, b, c, and d respectively with respect to some origin O.



Then, $\overrightarrow{AB} = \vec{b}-\vec{a}$, $\quad \overrightarrow{AD} = \vec{d}-\vec{a}$,

$\overrightarrow{BC} = \vec{c}-\vec{b}$, $\quad \overrightarrow{BD} = \vec{d}-\vec{b}$,

$\overrightarrow{CD} = \vec{d}-\vec{c}$, $\quad \overrightarrow{CA} = \vec{a}-\vec{c}$

Now, $|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}|$

$= |(\vec{b}-\vec{a})\times(\vec{d}-\vec{c})+(\vec{c}-\vec{b})\times(\vec{d}-\vec{a})+(\vec{a}-\vec{c})\times(\vec{d}-\vec{b})|$

$= |\vec{b}\times\vec{d}-\vec{a}\times\vec{d}-\vec{b}\times\vec{c}+\vec{a}\times\vec{c}+\vec{c}\times\vec{d}-\vec{c}\times\vec{a}$

$-\vec{b}\times\vec{d}+\vec{b}\times\vec{a}+\vec{a}\times\vec{d}-\vec{a}\times\vec{b}-\vec{c}\times\vec{d}+\vec{c}\times\vec{b}|$

$= |-\vec{b}\times\vec{c}+\vec{a}\times\vec{c}-\vec{c}\times\vec{a}+\vec{b}\times\vec{a}-\vec{a}\times\vec{b}+\vec{c}\times\vec{b}|$

$= 2|\vec{b}\times\vec{a}+\vec{c}\times\vec{b}+\vec{a}\times\vec{c}|$ ...(1)

Also Area of ΔABC is

$= \frac{1}{2}|\overrightarrow{BC}\times\overrightarrow{BA}| = \frac{1}{2}|(\vec{c}-\vec{b})\times(\vec{a}-\vec{b})|$

$= \frac{1}{2}|(\vec{c}\times\vec{a}-\vec{c}\times\vec{b}-\vec{b}\times\vec{a}+\vec{b}\times\vec{b}|$

$= \frac{1}{2}|-\vec{b}\times\vec{a}-\vec{c}\times\vec{b}-\vec{a}\times\vec{c}| = \frac{1}{2}|\vec{b}\times\vec{a}+\vec{c}\times\vec{b}+\vec{a}\times\vec{c}|$

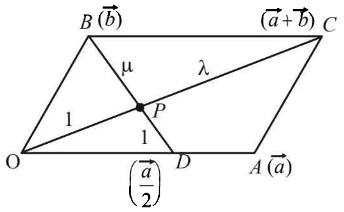
$\Rightarrow \quad 2Ar(\Delta ABC) = |\vec{b}\times\vec{a}+\vec{c}\times\vec{b}+\vec{a}\times\vec{c}|$ ...(2)

From (1) and (2), we get

$|\overrightarrow{AB}\times\overrightarrow{CD}+\overrightarrow{BC}\times\overrightarrow{AD}+\overrightarrow{CA}\times\overrightarrow{BD}|$

$= 2(2Ar(\Delta ABC)) = 4Ar(\Delta ABC)$ Hence Proved.

7. *OACB* is a parallelogram with *O* as origin. Let with respect to *O* position vectors of *A* and *B* be $\vec{a}$ and $\vec{b}$ respectively. Then p.v. of *C* is $\vec{a}+\vec{b}$.



Also *D* is mid pt. of *OA*, therefore position vector of *D* is $\frac{\vec{a}}{2}$.

CO and *BD* intersect each other at *P*.
Let *P* divides *CO* in the ratio λ : 1 and *BD* in the ratio μ : 1
Then by section theorem,
position vector of pt. *P* dividing *CO* in ratio

$$\lambda:1 = \frac{\lambda\times 0+1\times(\vec{a}+\vec{b})}{\lambda+1} = \frac{(\vec{a}+\vec{b})}{\lambda+1} \quad ...(1)$$

And position vector of pt. *P* dividing *BD* in the ratio μ : 1 is

$$= \frac{\mu\left(\frac{\vec{a}}{2}\right)+1(\vec{b})}{\mu+1} = \frac{\mu\vec{a}+2\vec{b}}{2(\mu+1)} \quad ...(2)$$

As (1) and (2) represent the position vector of same point, we should have

$$\frac{\vec{a}+\vec{b}}{\lambda+1} = \frac{\mu\vec{a}+2\vec{b}}{2(\mu+1)}$$

Equating the coefficients of $\vec{a}$ and $\vec{b}$, we get

$$\frac{1}{\lambda+1} = \frac{\mu}{2(\mu+1)} \quad ...(i)$$

$$\frac{1}{\lambda+1} = \frac{1}{\mu+1} \quad ...(ii)$$

From (ii) we get $\lambda=\mu \Rightarrow P$ divides *CO* and *BD* in the same ratio.
Putting $\lambda=\mu$ in eq. (i) we get $\mu=2$
Thus required ratio is 2 : 1.

8. Given that $\vec{a},\vec{b},\vec{c}$ are three coplanar vectors.
∴ There exists scalars *x*, *y*, *z*, not all zero, such that

$$x\vec{a}+y\vec{b}+z\vec{c}=\vec{0} \quad ...(1)$$

Taking dot product of $\vec{a}$ and (1), we get

$$x\vec{a}.\vec{a}+y\vec{a}.\vec{b}+z\vec{a}.\vec{c}=\vec{0} \quad ...(2)$$

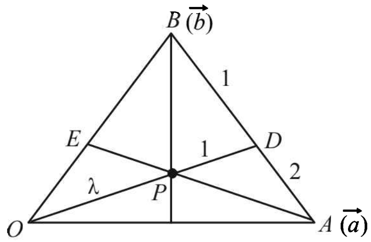
Again taking dot product of $\vec{b}$ and (1), we get

$$x\vec{b}.\vec{a}+y\vec{b}.\vec{b}+z\vec{b}.\vec{c}=\vec{0} \quad ...(3)$$

Now equations (1), (2), (3) form a homogeneous system of equations, where *x, y, z* are not all zero.
∴ system must have non trivial solution and for this, determinant of coefficient matrix should be zero

i.e. $$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \end{vmatrix} = 0$$ Hence Proved.

9. With *O* as origin let $\vec{a}$ and $\vec{b}$ be the position vectors of *A* and *B* respectively.



Then the position vector of *E*, the mid point of *OB* is $\frac{\vec{b}}{2}$.

Again since *AD* : *DB* = 2 : 1, the position vector of *D* is

$$\frac{1.\vec{a}+2\vec{b}}{1+2} = \frac{\vec{a}+2\vec{b}}{3}$$

∴ Equation of *OD* is

$$\vec{r}=t\left(\frac{\vec{a}+2\vec{b}}{3}\right) \quad ...(1)$$

and Equation of *AE* is

$$\vec{r}=\vec{a}+s\left(\frac{\vec{b}}{2}-\vec{a}\right) \quad ...(2)$$

If *OD* and *AE* intersect at *P*, then we will have identical values of $\vec{r}$. Hence comparing the coefficients of $\vec{a}$ and $\vec{b}$, we get

$$\frac{t}{3}=1-s \text{ and } \frac{2t}{3}=\frac{s}{2} \Rightarrow t=\frac{3}{5} \text{ and } s=\frac{4}{5}$$

Putting value of *t* in eq. (1) we get position vector of point of intersection *P* as

$$\frac{\vec{a}+2\vec{b}}{5} \quad ...(3)$$

Now if *P* divides *OD* in the ratio λ : 1, then p.v. of *P* is

$$\frac{\lambda\left(\frac{\vec{a}+2\vec{b}}{3}\right)+1.0}{\lambda+1} = \frac{\lambda}{3(\lambda+1)}(\vec{a}+2\vec{b}) \quad ...(4)$$

From (3) and (4) we get

$$\frac{\lambda}{3(\lambda+1)}=\frac{1}{5} \Rightarrow 5\lambda=3\lambda+3 \Rightarrow \lambda=3/2$$

∴ $OP:PD=3:2$

10. We are given that $\vec{A}=2\hat{i}+\hat{k}, \vec{B}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{C}=4\hat{i}-3\hat{j}+7\hat{k}$ and to determine a vector $\vec{R}$ such that $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$

Let $\vec{R}=x\hat{i}+y\hat{j}+z\hat{k}$

Then $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow \quad (y-z)\hat{i}-(x-z)\hat{j}+(x-y)\hat{k} = 10\hat{i}-11\hat{j}+7\hat{k}$

$\Rightarrow \quad y-z=-10 \qquad \ldots(1)$

$z-x=-11 \qquad \ldots(2)$

$x-y=7 \qquad \ldots(3)$

Also $\vec{R}.\vec{A}=0$

$\Rightarrow \quad 2x+z=0 \qquad \ldots(4)$

Substituting $y=x-7$ and $z=-2x$ from (3) and (4) respectively in eq. (1) we get

$x-7+2x=-10 \Rightarrow 3x=-3$

$\Rightarrow \quad x=-1,\ y=-8$ and $z=2$

$\therefore \quad \vec{R}=-\hat{i}-8\hat{j}+2\hat{k}$

11. We have, $\vec{a}=cx\hat{i}-6\hat{j}+3\hat{k}$, $\vec{b}=x\hat{i}-2\hat{j}+2cx\hat{k}$

Now we know that $\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

As angle between $\vec{a}$ and $\vec{b}$ is obtuse, therefore

$\cos\theta<0 \Rightarrow \vec{a}.\vec{b}<0$

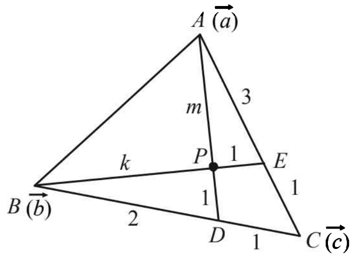
$\Rightarrow \quad cx^2-12+6cx<0 \Rightarrow -cx^2-6cx+12>0, \forall x\in R$

$\Rightarrow \quad -c>0$ and $D<0 \Rightarrow c<0$ and $36c^2+48c<0$

$\Rightarrow \quad c<0$ and $c(3c+4)<0 \Rightarrow c<0$ and $(3c+4)>0$

$\Rightarrow \quad c<0$ and $c>-4/3 \Rightarrow -4/3<c<0$

12. Let $\vec{a},\vec{b},\vec{c}$, be the position vectors of pt A, B and C respectively with respect to some origin.



ATQ, D divides BC in the ratio 2 : 1 and E divides AC in the ratio 3 : 1.

$\therefore$ position vector of D is $\dfrac{\vec{b}+2\vec{c}}{3}$ and position vector of E is $\dfrac{\vec{a}+3\vec{c}}{4}$

Let pt. of intersection P of AD and BE divides BE in the ratio k : 1 and AD in the ratio m : 1, then position vectors of P in these two cases are

$$\frac{\vec{b}+k\left(\frac{\vec{a}+3\vec{c}}{4}\right)}{k+1} \text{ and } \frac{\vec{a}+m\left(\frac{\vec{b}+2\vec{c}}{3}\right)}{m+1} \text{ respectively.}$$

Equating the position vectors of P in two cases we get

$$\frac{k}{4(k+1)}\vec{a}+\frac{1}{k+1}\vec{b}+\frac{3k}{4(k+1)}\vec{c}$$

$$=\frac{1}{m+1}\vec{a}+\frac{m}{3(m+1)}\vec{b}+\frac{2m}{3(m+1)}\vec{c}$$

$\Rightarrow \quad \dfrac{k}{4(k+1)}=\dfrac{1}{m+1} \qquad \ldots(1)$

$\dfrac{1}{k+1}=\dfrac{m}{3(m+1)} \qquad \ldots(2)$

$\dfrac{3k}{4(k+1)}=\dfrac{2m}{3(m+1)} \qquad \ldots(3)$

Dividing (3) by (2) we get

$\dfrac{3k}{4}=2 \Rightarrow k=\dfrac{8}{3} \Rightarrow$ the req. ratio is 8 : 3.

13. Given that $\vec{b},\vec{c},\vec{d}$ are not coplanar $\therefore [\vec{b},\vec{c},\vec{d}]\neq 0$

Consider, $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})$
$+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})$

Here, $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=-(\vec{c}\times\vec{d})\times(\vec{a}\times\vec{b})$

$=-(\vec{c}\times\vec{d}.\vec{b})\vec{a}+(\vec{c}\times\vec{d}.\vec{a})\vec{b}$

$=\left[\vec{a}\,\vec{c}\,\vec{d}\right]\vec{b}-\left[\vec{b}\,\vec{c}\,\vec{d}\right]\vec{a} \qquad \ldots(1)$

$(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})=-(\vec{d}\times\vec{b})\times(\vec{a}\times\vec{c})$

$=-(\vec{d}\times\vec{b}.\vec{c})\vec{a}+(\vec{d}\times\vec{b}.\vec{a})\vec{c}$

$=[\vec{a}\,\vec{d}\,\vec{b}]\vec{c}-[\vec{c}\,\vec{d}\,\vec{b}]\vec{a} \qquad \ldots(2)$

$(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})=\left((\vec{a}\times\vec{d})\vec{c}\right)\vec{b}-(\vec{a}\times\vec{d}.\vec{b})\vec{c}$

$=-[\vec{a}\,\vec{c}\,\vec{d}]\vec{b}-[\vec{a}\,\vec{d}\,\vec{b}]\vec{c} \qquad \ldots(3)$

[**NOTE :** Here we have tried to write the given expression in such a way that we can get terms involving $\vec{a}$ and other terms similar which can get cancelled.]

Adding (1), (2) and (3), we get

given vector $=-2[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}=k\vec{a}$

$\Rightarrow$ given vector = some constant multiple of $\vec{a}$

$\Rightarrow$ given vector is parallel to $\vec{a}$.

14. We are given $AD=4$

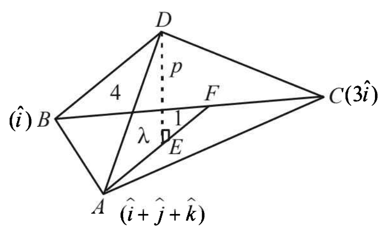
Volume of tetrahedron $=\dfrac{2\sqrt{2}}{3}$

$\Rightarrow \quad \dfrac{1}{3}Ar(\Delta ABC)p=\dfrac{2\sqrt{2}}{3}$

$\therefore \quad \dfrac{1}{2}|\overrightarrow{BA}\times\overrightarrow{BC}|\,p=2\sqrt{2}$

$\dfrac{1}{2}|(\hat{j}+\hat{k})\times 2i|\,p=2\sqrt{2}$ or $|\hat{j}-\hat{k}|\,p=2\sqrt{2}$

or $\quad \sqrt{2}p=2\sqrt{2} \quad \therefore p=2$



We have to find the P.V. of point E. Let it divides median AF in the ratio $\lambda : 1$

$\therefore$ P. V. of E is $\dfrac{\lambda.2\hat{i}+(\hat{i}+\hat{j}+\hat{k})}{\lambda+1}$ ...(2)

$\therefore$ $\overrightarrow{AE}=$ P.V. of $E-$P.V. of $A=\dfrac{\lambda}{\lambda+1}(\hat{i}-\hat{j}-\hat{k})$

$\therefore$ $|\overrightarrow{AE}|^2=\overrightarrow{AE}^2=\left(\dfrac{\lambda}{\lambda+1}\right)^2.3$ ...(3)

Now, $p^2+AE^2=AD^2$

or $4+\left(\dfrac{\lambda}{\lambda+1}\right)^2.3=16$ $\quad\therefore\quad 3\left(\dfrac{\lambda}{\lambda+1}\right)^2=12$

or $\left(\dfrac{\lambda}{\lambda+1}\right)=\pm 2$

$\lambda=\pm(2\lambda+2)$ $\quad\therefore\ \lambda=-2$ or $-2/3$

Putting the value of λ in (2) we get the P.V. of possible positions of E as $(-1, 3, 3)$ or $(3, -1, -1)$

15. We have, $(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})$

$=\vec{A}\times\vec{A}+\vec{B}\times\vec{A}+\vec{A}\times\vec{C}+\vec{B}\times\vec{C}$

$=\vec{B}\times\vec{A}+\vec{A}\times\vec{C}+\vec{B}\times\vec{C}$ $\quad[\because \vec{A}\times\vec{A}=0]$

Thus, $[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})]\times(\vec{B}\times\vec{C})$

$=[\vec{B}\times\vec{A}+\vec{A}\times\vec{C}+\vec{B}\times\vec{C}]\times(\vec{B}\times\vec{C})$

$=(\vec{B}\times\vec{A})\times(\vec{B}\times\vec{C})+(\vec{A}\times\vec{C})\times(\vec{B}\times\vec{C})$ $\quad[\because x\times x=0]$

$=\left\{(\vec{B}\times\vec{A}).\vec{C}\right\}\vec{B}-\left\{(\vec{B}\times\vec{A}).\vec{B}\right\}\vec{C}$

$+\left\{(\vec{A}\times\vec{C}).\vec{C}\right\}\vec{B}-\left\{(\vec{A}\times\vec{C}).\vec{B}\right\}\vec{C}$

$[\because\ (a\times b)\times c=(a.c)b-(b.c)a]$

$=[\vec{B}\vec{A}\vec{C}]\vec{B}-[\vec{A}\vec{C}\vec{B}]C$

[$\because$ [A B C]=0 if any two of A, B, C are equal.]

$=[\vec{A}\vec{C}\vec{B}]\left\{\vec{B}-\vec{C}\right\}$

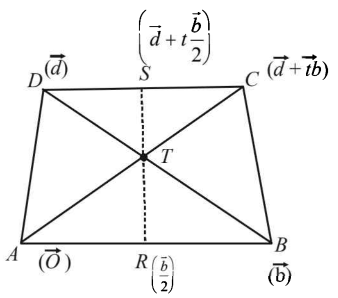
Thus, LHS of the given expression

$=[\vec{A}\vec{C}\vec{B}]\{(\vec{B}-\vec{C}).(\vec{B}+\vec{C})\}$

$=[\vec{A}\vec{C}\vec{B}]\{|\vec{B}|^2-|\vec{C}|^2\}=0$ $\quad[\because |B|=|C|]$

16. The P.Vs. of the points A, B, C, D are

$A(\vec{O}), B(\vec{b}), D(\vec{d}), C(\vec{d}+t\vec{b})$



Equations of AC and BD are

$r=\lambda(d+tb)$ and $r=(1-\mu)b+\mu d$

For point of intersection say T compare the coefficients

$\lambda=\mu,\ t\lambda=1-\mu=1-\lambda$ or $(t+1)\lambda=1$

$\therefore\quad \lambda=\dfrac{1}{t+1}=\mu$

$\therefore\quad T$ is $\dfrac{d+tb}{t+1}$ ...(1)

Let R and S be mid-points of parallel sides AB and DC then

R is $\dfrac{b}{2}$ and S is $d+t\dfrac{b}{2}$.

Equation of RS by $r=a+s(b-a)$ is

$r=\dfrac{b}{2}+s\left[d+(t-1)\dfrac{b}{2}\right]$

The point (1) will lie on above if,

$\dfrac{d+tb}{1+t}=\dfrac{b}{2}+s\left[d+(t-1)\dfrac{b}{2}\right]$

Comparing the coefficients, we get

$\dfrac{t}{1+t}=\dfrac{1}{2}+s\dfrac{(t-1)}{2}$ and $\dfrac{t}{(1+t)}=s,$

$\therefore\quad \dfrac{t}{1+t}=\dfrac{1}{2}+\dfrac{1}{1+t}.\dfrac{(t-1)}{2}=\dfrac{2t}{2(1+t)}=\dfrac{t}{1+t}$

which is true . Hence proved.

17. (a) We have, $\vec{u}.\vec{v}=|\vec{u}||\vec{v}|\cos\theta$ and $\vec{u}\times\vec{v}=|\vec{u}||\vec{v}|\sin\theta\,\hat{n}$

Where θ is the angle between $\vec{u}$ and $\vec{v}$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{u},\vec{v}$ and is such that $\vec{u},\vec{v},\hat{n}$ form a right handed system.

Thus, $|\vec{u}.\vec{v}|^2=|\vec{u}|^2|\vec{v}|^2\cos^2\theta$

and $|\vec{u}\times\vec{v}|^2=|\vec{u}|^2|\vec{v}|^2\sin^2\theta\,\hat{n}.\hat{n}=|\vec{u}|^2|\vec{v}|^2\sin^2\theta$

$\therefore\quad |u.v|^2+|u\times v|^2=|u|^2|v|^2(\cos^2\theta+\sin^2\theta)$
$=|u|^2|v|^2$

(b) Let $|u|=a$, $|v|=b$, $u\times v=ab\sin\theta\,\hat{n}$, where $\hat{n}$ is perpendicular to both u and v, $|a|^2=a^2$

L.H.S. $=(1+a^2)\ (1+b^2)$

R.H.S. $=(1-ab\cos\theta)^2+(u+v)^2\times(u\times v)^2$
$+2(u+v).ab\sin\theta\,\hat{n}$

$=1+a^2b^2\cos^2\theta-2ab\cos\theta+a^2$
$+b^2+2ab\cos\theta+a^2b^2\sin^2\theta.1+0$

as $\hat{n}$ is $\perp$ to both $\boldsymbol{u}$ and $\boldsymbol{v}$.

$=1+a^2b^2(\cos^2\theta+\sin^2\theta)+a^2+b^2$

$=1+a^2+b^2+a^2b^2=(1+a^2)(1+b^2)$

18. $[\vec{u}\,\vec{v}\,\vec{w}] = (\vec{u}\times\vec{v}).(\vec{v}-\vec{w}\times\vec{u}) = (\vec{u}\times\vec{v}).(\vec{u}\times\vec{w})$

$$= \begin{vmatrix} \vec{u}.\vec{u} & \vec{u}.\vec{w} \\ \vec{v}.\vec{u} & \vec{v}.\vec{w} \end{vmatrix}$$

Now, $\vec{u}.\vec{u} = 1$

$\vec{u}.\vec{w} = \vec{u}.(\vec{v}-\vec{w}\times\vec{u}) = \vec{u}.\vec{v} - [\vec{u}\,\vec{w}\,\vec{u}] = \vec{u}.\vec{v}$

$\vec{v}.\vec{w} = \vec{v}.(\vec{v}-\vec{w}\times\vec{u}) = 1-[\vec{v}\,\vec{w}\,\vec{u}] = 1-[\vec{u}\,\vec{v}\,\vec{w}]$

$\therefore \quad [\vec{u}\,\vec{v}\,\vec{w}] = \begin{vmatrix} 1 & \cos\theta \\ \cos\theta & 1-(\vec{u}\,\vec{v}\,\vec{w}) \end{vmatrix},$

(θ is angle between $\vec{u}$ and $\vec{v}$)

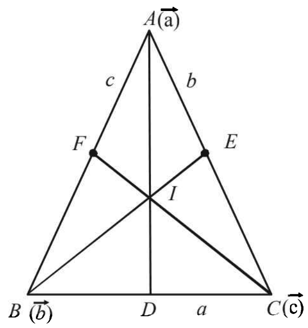
$$= 1-[\vec{u}\,\vec{v}\,\vec{w}] - \cos^2\theta$$

$\therefore \quad [\vec{u}\,\vec{v}\,\vec{w}] = \frac{1}{2}\sin^2\theta \le \frac{1}{2}$

Equality holds when $\sin^2\theta = 1$ i.e., $\theta = \pi/2 \therefore \vec{u} \perp \vec{v}$.

19. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors by A, B, and C respectively,

Let AB, BE and CF be the bisectors of $\angle A$, $\angle B$, and $\angle C$ respectively.



a, b, c are the lengths of sides BC, CA and AB respectively. Now we know by angle bisector thm that AD divides, BC in the ratio

$$BD : DC = AB : AC = c : b.$$

$\therefore$ The position vector of D is $\vec{d} = \dfrac{b\vec{b}+c\vec{c}}{b+c}$

Let I be the point of intersection of BE and AD. Then in ΔABD, BI is bisector of $\angle B$.

$\therefore$ DI : IA = BD : BA

But $\dfrac{BD}{DC} = \dfrac{c}{b} \Rightarrow \dfrac{BD}{BD+DC} = \dfrac{c}{c+b}$

$\Rightarrow \dfrac{BD}{BC} = \dfrac{c}{c+b} \Rightarrow BD = \dfrac{ac}{b+c}$

$\therefore \quad DI : IA = \dfrac{ac}{b+c} : c = a : (b+c)$

$\therefore$ P.V. of $I = \dfrac{\vec{a}.\, a + \vec{d}(b+c)}{a+b+c}$

$$= \frac{a\vec{a} + \left(\dfrac{b\vec{b}+c\vec{c}}{b+c}\right)(b+c)}{a+b+c} = \frac{a\vec{a}+b\vec{b}+c\vec{c}}{a+b+c}$$

As p.v. of I is symm. in $\vec{a}, \vec{b}, \vec{c}$ and a,b,c.

$\therefore$ It must lie on CF as well.

[We can also see that p.v. of intersection of AD and CF is also $\dfrac{a\vec{a}+b\vec{b}+c\vec{c}}{a+b+c}$]

Above prove that all the $\angle$ bisectors pass through I, i.e., these are concurrent.

20. Given data is insufficient to uniquely determine the three vectors as there are only 6 equations involving 9 variables.

$\therefore$ We can obtain infinitely many set of three vectors, $\vec{v}_1, \vec{v}_2, \vec{v}_3$, satisfying these conditions.

From the given data, we get

$\vec{v}_1.\vec{v}_1 = 4 \Rightarrow |\vec{v}_1| = 2$

$\vec{v}_2.\vec{v}_2 = 2 \Rightarrow |\vec{v}_2| = \sqrt{2}$

$\vec{v}_3.\vec{v}_3 = 29 \Rightarrow |\vec{v}_3| = \sqrt{29}$

Also $\vec{v}_1.\vec{v}_2 = -2 \Rightarrow |\vec{v}_1|\,|\vec{v}_2|\cos\theta = -2$

[where θ is the angle between $\vec{v}_1$ and $\vec{v}_2$]

$\Rightarrow \quad \cos\theta = \dfrac{-1}{\sqrt{2}} \Rightarrow \theta = 135°$

Now since any two vectors are always coplanar, let us suppose that $\vec{v}_1$ and $\vec{v}_2$ are in x-y plane.

Let $\vec{v}_1$ is along the positive direction of x-axis

then $\vec{v}_1 = 2\hat{i}$. $\quad [\because |\vec{v}_1| = 2]$

As $\vec{v}_2$ makes an angle 135° with $\vec{v}_1$ and lines in x-y plane,

Also keeping in mind $|\vec{v}_2| = \sqrt{2}$, we obtain $\vec{v}_2 = -\hat{i} \pm \hat{j}$

Again let $\vec{v}_3 = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\because \quad \vec{v}_3.\vec{v}_1 = 6 \Rightarrow 2\alpha = 6 \Rightarrow \alpha = 3$

and $\vec{v}_3.\vec{v}_2 = -5 \Rightarrow -\alpha \pm \beta = -5 \Rightarrow \beta = \pm 2$

Also $|\vec{v}_3| = \sqrt{29} \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 29 \Rightarrow \gamma = \pm 4$

Hence $\vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$

Thus, $\vec{v}_1 = 2\hat{i}$; $\vec{v}_2 = -\hat{i} \pm \hat{j}$; $\vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$ are some possible answers.

21. $\vec{A}(t)$ is parallel to $\vec{B}(t)$ for some $t \in [0,1]$ if and only if

$\dfrac{f_1(t)}{g_1(t)} = \dfrac{f_2(t)}{g_2(t)}$ for some $t \in [0,1]$

or $f_1(t).g_2(t) = f_2(t).g_1(t)$ for some $t \in [0,1]$

Let $\quad h(t) = f_1(t).g_2(t) - f_2(t).g_1(t)$

$h(0) = f_1(0).g_2(0) - f_2(0).g_1(0)$

$= 2\times 2 - 3\times 3 = -5 < 0$

$h(1) = f_1(1).g_2(1) - f_2(1).g_1(1)$

$= 6\times 6 - 2\times 2 = 32 > 0$

Since h is a continuous function, and $h(0).h(1) < 0$

$\Rightarrow$ There is some $t \in [0,1]$ for which $h(t)=0$ i.e., $\vec{A}(t)$ and $\vec{B}(t)$ are parallel vectors for this t.

22. $\therefore$ We have $V = [\vec{a}\ \vec{b}\ \vec{c}\,] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$$\Rightarrow V = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1) \quad ....(1)$$

Now we know that $AM \geq GM$

$$\therefore \frac{(a_1+b_1+c_1)+(a_2+b_2+c_2)+(a_3+b_3+c_3)}{3}$$

$$\geq [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/3}$$

$$\Rightarrow \frac{3L}{3} \geq [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/3}$$

$$\Rightarrow L^3 \geq (a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)$$

$\Rightarrow L^3 \geq a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 + 24$ more such terms

$L^3 \geq a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2$ $[\because a_r, b_r, c_r \geq 0 \text{ for } r = 1,2,3]$

$L^3 \geq (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$ [same reason]

$L^3 \geq V$ from (1) Hence Proved.

23. (i) Plane passing through (2, 1, 0), (5, 0, 1) and (4, 1, 1) is

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 5-2 & 0-1 & 1-0 \\ 4-2 & 1-1 & 1-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 3 & -1 & 1 \\ 2 & 0 & 1 \end{vmatrix} = 0$$

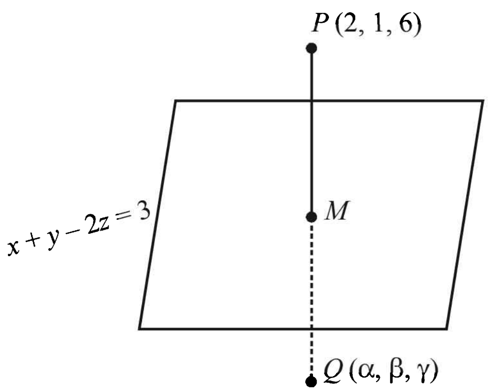
$\Rightarrow$ $(x-2)(-1-0)-(y-1)(3-2)+z(0-(-2))=0$

$\Rightarrow$ $-x+2-y+1+2z=0$ $\Rightarrow$ $x+y-2z=3$

(ii) As per question we have to find a pt. Q such that PQ is $\perp$ to the plane $x+y-2z=3$ ...(1)

And mid pt. of PQ lies on the plane, (Clearly we have to find image of pt. P with respect to plane).

Let Q be (α, β, γ)



Eq^n of PM passing through $P(2, 1, 6)$ and $\perp$ to plane $x+y-2z=3$, is given by

$$\frac{x-2}{1} = \frac{y-1}{1} = \frac{z-6}{-2} = \lambda$$

For some value of λ, Q (α, β, γ) lies on PM

$$\therefore \quad \frac{\alpha-2}{1} = \frac{\beta-1}{1} = \frac{\gamma-6}{-2} = \lambda$$

$\Rightarrow$ $\alpha = \lambda+2$, $\beta = \lambda+1$, $\gamma = -2\lambda+6$

$\therefore$ Mid. pt. of PQ

i.e. $M\left(\frac{2+\lambda+2}{2}, \frac{1+\lambda+1}{2}, \frac{6-2\lambda+6}{2}\right)$

$$= \left(\frac{\lambda+4}{2}, \frac{\lambda+2}{2}, \frac{12-2\lambda}{2}\right)$$

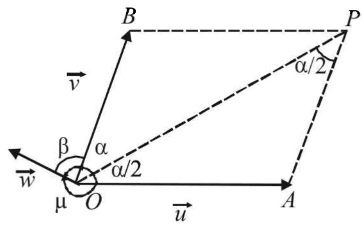
But M lies on plane (1)

$$\therefore \quad \frac{\lambda+4}{2} + \frac{\lambda+2}{2} - 12 - 2\lambda = 3$$

$\Rightarrow \lambda+4+\lambda+2-24+4\lambda = 6 \Rightarrow 6\lambda = 24 \Rightarrow \lambda = 4$

$\therefore$ $Q(4+2, 4+1, -8+6) = (6, 5, -2)$

24. Given that $\vec{u}, \vec{v}, \vec{\omega}$ are three non coplanar unit vectors. Angle between $\vec{u}$ and $\vec{v}$ is α, between $\vec{v}$ and $\vec{\omega}$ is β and between $\vec{\omega}$ and $\vec{u}$ it is γ. In fig. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent $\vec{u}$ and $\vec{v}$. Let P be a pt. on angle bisector of $\angle AOB$ such that $OAPB$ is a parallelogram.



Also $\angle POA = \angle BOP = \alpha/2$

$\therefore$ $\angle APO = \angle BOP = \alpha/2$ (alt. int. $\angle$'s)

$\therefore$ In ΔOAP, $OA = AP$

a unit vector in the direction of $\overrightarrow{OP}$

$\therefore$ $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \vec{u} + \vec{v}$

$\therefore$ A unit vector in the direction of

$$\overrightarrow{OP} = \frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|} \quad \text{i.e. } \vec{x} = \frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}$$

But $|\vec{u}+\vec{v}|^2 = (\vec{u}+\vec{v}).(\vec{u}+\vec{v}) = 1+1+2\vec{u}.\vec{v}$ $[\because |\vec{u}| = |\vec{v}| = 1]$

$$= 2 + 2\cos\alpha = 4\cos^2\alpha/2$$

$$\therefore \quad |\vec{u}+\vec{v}| = 2\cos\alpha/2 \Rightarrow \vec{x} = \frac{1}{2}(\sec\alpha/2)(\vec{u}+\vec{v})$$

Similarly, $\vec{y} = \frac{1}{2}\sec\frac{\beta}{2}(\vec{v}+\vec{\omega})$ and $\vec{z} = \frac{1}{2}\sec\frac{\gamma}{2}(\vec{\omega}+\vec{u})$

Now consider $[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}]$

$$= (\vec{x}\times\vec{y}).[(\vec{y}\times\vec{z})\times(\vec{z}\times\vec{x})]$$

$$= (\vec{x}\times\vec{y}).[\{(\vec{y}\times\vec{z}).\vec{x}\}\vec{z} - \{(\vec{y}\times\vec{z}).\vec{z}\}\vec{x}]$$

[Using def^n of vector triple product.]

$= (\vec{x}\times\vec{y}).[[\vec{x}\,\vec{y}\,\vec{z}]\vec{z} - 0]$ $[\therefore [\vec{y}\,\vec{z}\,\vec{z}] = 0]$

$=[\vec{x}\vec{y}\vec{z}][\vec{x}\vec{y}\vec{z}]=[\vec{x}\vec{y}\vec{z}]^2$ ...(1)

Also $[\vec{x}\vec{y}\vec{z}]=\left[\frac{1}{2}\left(\sec\frac{\alpha}{2}\right)(\vec{u}+\vec{v})\frac{1}{2}\sec\beta/2\right.$

$\left.(\vec{v}+\vec{\omega})\frac{1}{2}\sec\gamma/2(\vec{\omega}+\vec{u})\right]$

$=\frac{1}{8}\sec\alpha/2\sec\beta/2\sec\gamma/2[\vec{u}+\vec{v}\ \vec{v}+\vec{\omega}\ \vec{\omega}+\vec{u}]$

$=\frac{1}{8}\sec\alpha/2\sec\beta/2\sec\gamma/2[(\vec{u}+\vec{v}).\{(\vec{v}+\vec{\omega})\times(\vec{\omega}+\vec{u})\}]$

$=\frac{1}{8}\sec\alpha/2\sec\beta/2\sec\gamma/2[(\vec{u}+\vec{v}).(\vec{v}\times\vec{\omega}+\vec{v}\times\vec{u}+\vec{\omega}\times\vec{u})]$

$=\frac{1}{8}\sec\alpha/2\sec\beta/2\sec\gamma/2[\vec{u}.(\vec{v}\times\vec{\omega})+\vec{v}.(\vec{\omega}\times\vec{u})]$

($\because$ $[\vec{a}\vec{b}\vec{c}]=0$ when ever any two vectors are same)

$=\frac{1}{8}(\sec\alpha/2\sec\beta/2\sec\gamma/2)2[\vec{u}\vec{v}\vec{\omega}]$

$=\frac{1}{4}(\sec\alpha/2\sec\beta/2\sec\gamma/2)[\vec{u}\vec{v}\vec{\omega}]$

$\therefore [\vec{x}\vec{y}\vec{z}]^2=\frac{1}{16}[\vec{u}\vec{v}\vec{\omega}]^2\sec^2\alpha/2\ \sec^2\beta/2\ \sec^2\gamma/2]$ ...(2)

From (1) and (2),

$[\vec{x}\times\vec{y}\ \vec{y}\times\vec{z}\ \vec{z}\times\vec{x}]=\frac{1}{16}[\vec{u}\vec{v}\vec{\omega}]^2\sec^2\frac{\alpha}{2}.\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$

25. Given that $\vec{a}\neq\vec{b}\neq\vec{c}\neq\vec{d}$

Such that $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ ...(1)

$\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ ...(2)

To prove $(\vec{a}-\vec{d}).(\vec{b}-\vec{c})\neq 0$

Subtracting eq^n (2) from (1) we get

$\vec{a}\times(\vec{c}-\vec{b})=(\vec{b}-\vec{c})\times\vec{d}\ \Rightarrow\ \vec{a}\times(\vec{c}-\vec{b})=\vec{d}\times(\vec{c}-\vec{b})$

$\Rightarrow\ \vec{a}\times(\vec{c}-\vec{b})-\vec{d}\times(\vec{c}-\vec{b})=0$

$\Rightarrow\ (\vec{a}-\vec{d})\times(\vec{c}-\vec{b})=0\ \Rightarrow\ (\vec{a}-\vec{d})\,||\,(\vec{c}-\vec{b})$

[$\because$ $\vec{a}-\vec{d}\neq 0, \vec{c}-\vec{b}\neq 0$ as all distinct]

$\Rightarrow$ Angle between $\vec{a}-\vec{d}$ and $\vec{c}-\vec{b}$ is either 0 or $180°$.

$\Rightarrow\ (\vec{a}-\vec{d}).(\vec{c}-\vec{b})=|\vec{a}-\vec{d}||\vec{c}-\vec{b}|\cos 0\,[\text{or } \cos 180°]\neq 0$

as *a, b, c, d* all are different. Hence Proved.

26. $\because$ The plane is parallel to the lines L_1 and L_2 with direction ratios as $(1, 0, -1)$ and $(1, -1, 0)$ $\therefore$

A vector perpendicular to L_1 and L_2 will be parallel to the normal $(\vec{n})$ to the plane.

$$\therefore\ \vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 0 & -1\\ 1 & -1 & 0\end{vmatrix}=-\hat{i}-\hat{j}-\hat{k}$$

$\therefore$ Eqn. of plane through (1, 1, 1) and having normal vector $\vec{n}=-\hat{i}-\hat{j}-\hat{k}$ is given by $(\vec{r}-\vec{a}).\vec{n}=0$

$\Rightarrow\ -1(x-1)-1(y-1)-1(z-1)=0\ \Rightarrow\ x+y+z=3$

$\Rightarrow\ \frac{x}{3}+\frac{y}{3}+\frac{z}{3}=1$ ...(1)

Now the pts where this plane meets the axes are $A(3,0,0), B(0,3,0), C(0,0,3)$

$\therefore$ Vol.of tetrahedron *OABC*

$=\frac{1}{6}\times$ Area of base $\times$ altitude

$=\frac{1}{6}\times$ Ar$(\Delta ABC)\times$ length of $\perp^{\text{lar}}$ $(0,0,0)$ to plane (1)

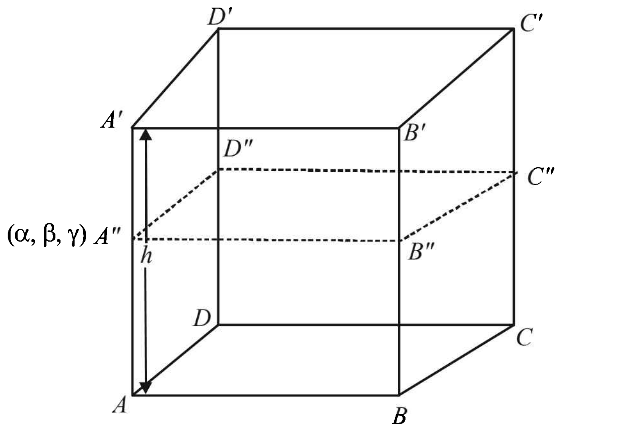
$=\frac{1}{6}\times\frac{1}{2}\left[\frac{\sqrt{3}}{4}\times|\overrightarrow{AB}|^2\right]\times\left[\left|\frac{-3}{\sqrt{1+1+1}}\right|\right]$

(Note that ΔABC is an equilateral Δ here.)

$=\frac{1}{12}\times\frac{\sqrt{3}}{4}\times(3\sqrt{2})^2\times\sqrt{3}\ =\frac{3\times 18}{48}=\frac{9}{2}$ cubic units.

27. ATQ 'S' is the parallelopiped with base points *A, B, C* and *D* and upper face points A', B', C' and D'. Let its vol. be V_S.

By compressing it by upper face A', B', C', D', a new parallelopiped 'T' is formed whose upper face pts are now A'', B'', C'' and D''. Let its vol. be V_T.



Let *h* be the height of original parallelopiped *S*.

Then $V_S=(ar\ ABCD)\times h$ ...(1)

Let equation of plane *ABCD* be

$ax+by+cz+d=0$ and $A''(\alpha,\beta,\gamma)$

Then height of new parallelopipe *T* is the length of perpendicular from A'' to *ABCD*

i.e. $\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}$

$\therefore\ V_T=(ar\ ABCD)\times\frac{a\alpha+b\beta+c\gamma+d}{\sqrt{a^2+b^2+c^2}}$ ...(2)

But given that,

$$V_T = \frac{90}{100} V_s \qquad ...(3)$$

From (1), (2) and (3) we get,

$$\frac{a\alpha + b\beta + c\gamma + d}{\sqrt{a^2+b^2+c^2}} = 0.9h$$

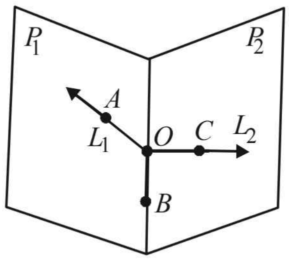
$$\Rightarrow \quad a\alpha + b\beta + c\gamma + (d - 0.9h\sqrt{a^2+b^2+c^2}) = 0$$

$\therefore$ Locus of $A''(\alpha, \beta, \gamma)$ is

$$ax + by + cz + (d - 0.9h\sqrt{a^2+b^2+c^2}) = 0$$

which is a plane parallel to $ABCD$. Hence proved.

28. Following fig. shows the possible situation for planes P_1 and P_2 and the lines L_1 and L_2



Now if we choose pts *A*, *B*, *C* as follows.

A on L_1, *B* on the line of intersection of P_1 and P_2 but other than origin and *C* on L_2 again other than origin then we can consider

A corresponds to one of A', B', C' and

B corresponds to one of the remaining of A', B', C' and

C corresponds to third of A', B', C' e.g.

$A' \equiv C; B' \equiv B; C' \equiv A$

Hence one permutation of $[ABC]$ is $[CBA]$. Hence Proved.

29. The given line is $2x - y + z - 3 = 0 = 3x + y + z - 5$

Which is intersection line of two planes

$$2x - y + z - 3 = 0 \qquad ...(i)$$

$$\text{and } 3x + y + z - 5 = 0 \qquad ...(ii)$$

Any plane containing this line will be the plane passing through the intersection of two planes (i) and (ii).

Thus the plane containing given line can be written as

$$(2x - y + z - 3) + \lambda(3x + y + z - 5) = 0$$

$$\Rightarrow \quad (3\lambda + 2)x + (\lambda - 1)y + (\lambda + 1)z + (-5\lambda - 3) = 0$$

As its distance from the pt. (2, 1, – 1) is $\frac{1}{\sqrt{6}}$

$$\therefore \quad \left| \frac{(3\lambda+2)2 + (\lambda-1)1 + (\lambda+1)(-1) + (-5\lambda-3)}{\sqrt{(3\lambda+2)^2 + (\lambda-1)^2 + (\lambda+1)^2}} \right| = \frac{1}{\sqrt{6}}$$

$$\Rightarrow \quad \left| \frac{\lambda - 1}{\sqrt{11\lambda^2 + 12\lambda + 6}} \right| = \frac{1}{\sqrt{6}}$$

Squaring both sides, we get

$$\frac{(\lambda-1)^2}{11\lambda^2 + 12\lambda + 6} = \frac{1}{6}$$

$$\Rightarrow \quad 6\lambda^2 - 12\lambda + 6 - 11\lambda^2 - 12\lambda - 6 = 0$$

$$\Rightarrow \quad 5\lambda^2 + 24\lambda = 0 \Rightarrow \lambda(5\lambda + 24) = 0$$

$$\Rightarrow \quad \lambda = 0 \text{ or } -24/5$$

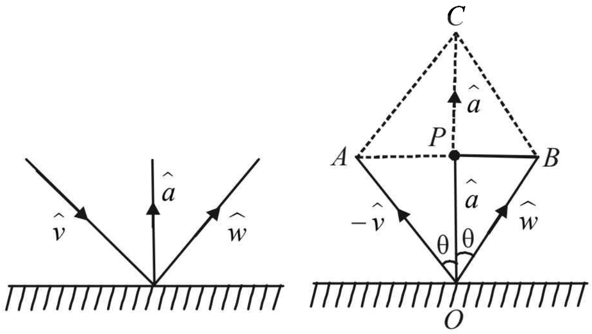
$\therefore$ The required equations of planes are

$$2x - y + z - 3 = 0$$

$$\text{and } \left[3\left(\frac{-24}{5}\right) + 2\right]x + \left[-\frac{24}{5} - 1\right]y + \left[-\frac{24}{5} + 1\right]z - 5\left(\frac{-24}{5}\right) - 3 = 0$$

$$\text{or} \quad 62x + 29y + 19z - 105 = 0$$

30. Given that incident ray is along $\hat{v}$, reflected ray is along $\hat{w}$ and normal is along $\hat{a}$, outwards. The given figure can be redrawn as shown.



We know that incident ray, reflected ray and normal lie in a plane, and angle of incidence = angle of reflection.

Therefore $\hat{a}$ will be along the angle bisector of $\hat{w}$ and $-\hat{v}$,

$$\text{i.e.,} \quad \hat{a} = \frac{\hat{w} + (-\hat{v})}{|\hat{w} - \hat{v}|} \qquad ...(1)$$

[$\because$ Angle bisector will along a vector dividing in same ratio as the ratio of the sides forming that angle.]

But $\hat{a}$ is a unit vector.

Where $|\hat{w} - \hat{v}| = OC = 2OP = 2|\hat{w}|\cos\theta = 2\cos\theta$

Substituting this value in equation (1) we get

$$\hat{a} = \frac{\hat{w} - \hat{v}}{2\cos\theta}$$

$$\therefore \quad \hat{w} = \hat{v} + (2\cos\theta)\hat{a} = \hat{v} - 2(\hat{a}.\hat{v})\hat{a} \quad [\because \hat{a}.\hat{v} = -\cos\theta]$$

F. Match the Following

1. **(A)→(s); (B)→(p); (C)→(q), (r); (D)→(s)**

(A) On solving the given equations $x + y = |a|$ and $ax - y = 1$, we get

$$x = \frac{1 + |a|}{a + 1} \text{ and } y = \frac{a|a| - 1}{a + 1}$$

$\because$ Rays intersect each other in I quad.

$\therefore x, y > 0 \Rightarrow a+1 > 0$ and $a|a| - 1 > 0 \Rightarrow a > 1$
$\therefore a_0 = 1$(A) $\rightarrow$ (s)

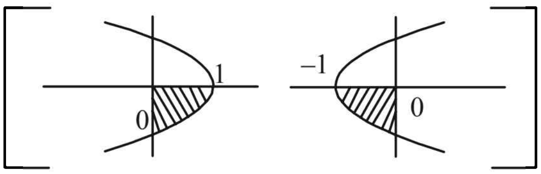
(B) (α, β, γ) lies on the plane $x+y+z=2$
$\Rightarrow \alpha+\beta+\gamma=2$

Also $\hat{k} \times (\hat{k} \times \vec{a}) = (\hat{k}.\vec{a})\hat{k} - (\hat{k}.\hat{k})\vec{a}$

$\Rightarrow \gamma\hat{k} - \alpha\hat{i} - \beta\hat{j} - \gamma\hat{k} = 0 \Rightarrow \alpha\hat{i} + \beta\hat{j} = 0$
$\Rightarrow \alpha = 0 = \beta \Rightarrow \gamma = 2$ $\quad (\because \alpha+\beta+\gamma=2)$
(B) $\rightarrow$ (p)

(C) $\left|\int_0^1 (1-y^2)dy\right| + \left|\int_0^1 (y^2-1)dy\right| = 2\int_0^1 (1-y^2)dy = \frac{4}{3}$

Also, $\left|\int_0^1 \sqrt{1-x}\,dx\right| + \int_{-1}^0 \sqrt{1+x}\,dx = 2\int_0^1 \sqrt{1-x}\,dx$



$[\because y = \sqrt{1-x}$, i.e., $y^2 = -(x-1)$ and $y = \sqrt{1+x}$
i.e., $y^2 = (x+1)$ represent same area under the given limits]

$= 2\int_0^1 \sqrt{x}\,dx \left[\text{Using} \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \left[2.\frac{2}{3}x^{3/2}\right]_0^1 = \frac{4}{3}$, $\quad$ (C) $\rightarrow$ (r) and (q)

(D) Given : $\sin A \sin B \sin C + \cos A \cos B = 1$
But $\sin A \sin B \sin C + \cos A \cos B \le \sin A \sin B + \cos A \cos B = \cos(A-B)$
$\Rightarrow \cos(A-B) \ge 1 \Rightarrow \cos(A-B) = 1$
$\Rightarrow A - B = 0 \Rightarrow A = B$
$\therefore$ Given relation becomes $\sin^2 A \sin C + \cos^2 A = 1$
$\Rightarrow \sin C = 1$,
(D) $\rightarrow$ (s)

2. **(A) $\rightarrow$ r ; (B) $\rightarrow$ q; (C) $\rightarrow$ p ; (D) $\rightarrow$ s**
Here we have, the determinant of the coefficient matrix of given equation, as

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

(A) $a+b+c \ne 0$ and $a^2+b^2+c^2-ab-bc-ca = 0$
$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2 = 0$
$\Rightarrow a = b = c$ $\quad$ (but $\ne 0$ as $a+b+c \ne 0$)
This equation represent identical planes.

(B) $a+b+c = 0$ and $a^2+b^2+c^2-ab-bc-ca \ne 0$
$\Rightarrow \Delta = 0$ and a, b, c are not all equal.
$\therefore$ All equations are not identical but have infinite many solutions.

$\therefore ax+by = (a+b)z$ $\quad$ (using $a+b+c=0$)
and $bx+cy = (b+c)z$
$\Rightarrow (b^2-ac)y = (b^2-ac)z \quad \Rightarrow y=z$
$\Rightarrow ax+by+cy = 0 \Rightarrow ax = ay \Rightarrow x = y$
$\Rightarrow x = y = z$
$\therefore$ The equations represent the line $x = y = z$

(C) $a+b+c \ne 0$ and $a^2+b^2+c^2-ab-bc-ca \ne 0$
$\Rightarrow \Delta \ne 0 \Rightarrow$ Equations have only trivial solution i.e., $x = y = z = 0$
$\therefore$ the equations represents the three planes meeting at a single point namely origin.

(D) $a+b+c = 0$ and $a^2+b^2+c^2-ab-bc-ca = 0$
$\Rightarrow a = b = c$ and $\Delta = 0 \Rightarrow a = b = c = 0$
$\Rightarrow$ All equations are satisfied by all x, y, and z.
$\Rightarrow$ The equations represent the whole of the three dimensional space (all points in 3–D)

3. **A $\rightarrow$ q,s; B $\rightarrow$ p,r,s,t; C $\rightarrow$ t; D $\rightarrow$ r**
(A) The given equation is

$2\sin^2\theta + \sin^2 2\theta = 2$
$\Rightarrow 2\sin^2\theta + 4\sin^2\theta\cos^2\theta - 2 = 0$
$\Rightarrow \sin^2\theta + 2\sin^2\theta(1-\sin^2\theta) - 1 = 0$
$\Rightarrow 2\sin^4\theta - 3\sin^2\theta + 1 = 0$
$\Rightarrow 2\sin^4\theta - 2\sin^2\theta - \sin^2\theta + 1 = 0$
$\Rightarrow 2\sin^2\theta(\sin^2\theta - 1) - 1(\sin^2\theta - 1) = 0$
$\Rightarrow (\sin^2\theta - 1)(2\sin^2\theta - 1) = 0$
$\Rightarrow \sin^2\theta = 1$ or $\sin^2\theta = \frac{1}{2}$
$\Rightarrow \sin^2\theta = \sin^2\frac{\pi}{2}$ or $\sin^2\theta = \sin^2\frac{\pi}{4}$
$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}$ or $n\pi \pm \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2}$ or $\frac{\pi}{4}$

(B) We know that $[x]$ is discontinuous at all integral values, therefore $\left[\frac{6x}{\pi}\right]$ is discontinuous at $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and π. Also $\cos\left[\frac{3x}{\pi}\right] \ne 0$ for any of these values of x.

$\therefore \left[\frac{6x}{\pi}\right]\cos\left[\frac{3x}{\pi}\right]$ is discontinuous at $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ and π.

(C) We know that the volume of a parallelopipe with coterminus edges as $\vec{a}, \vec{b}$ and $\vec{c}$ is given by $[\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore$ The required volume is $= \vec{a}.\vec{b} \times \vec{c}$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi$$

(D) We have $\vec{a}+\vec{b}=-\sqrt{3}\vec{c} \Rightarrow |\vec{a}+\vec{b}|^2=3|\vec{c}|^2$

$\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b})=3\vec{c}.\vec{c}$

$\Rightarrow \vec{a}.\vec{a}+\vec{b}.\vec{b}+2\vec{a}.\vec{b}=3\vec{c}.\vec{c} \Rightarrow 1+1+2\cos\theta=3$

(where θ is the angle between $\vec{a}$ and $\vec{b}$)

$\Rightarrow \cos\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

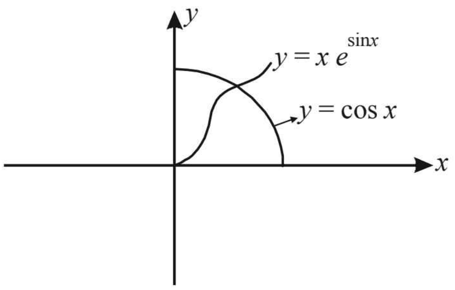
4. $\mathbf{A\to p;\ B\to q,s;\ C\to q,r,s,t;\ D\to r}$

(A) For the solution of $xe^{\sin x}-\cos x=0$ in $\left(0,\frac{\pi}{2}\right)$

Let us consider two functions

$y=xe^{\sin x}$ and $y=\cos x$

The range of $y=xe^{\sin x}$ is $\left(0,\frac{\pi e}{2}\right)$, also it is an increasing function on $\left(0,\frac{\pi}{2}\right)$. Their graph are as shown in the figure below :



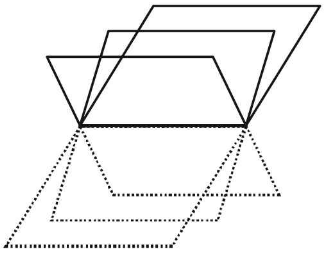
Clearly the two curves meet only at one point, therefore the given equation has only one solution in $\left(0,\frac{\pi}{2}\right)$.

(B) Three given planes are

$kx+4y+z=0$

$4x+ky+2z=0$

$2x+2y+z=0$



Clearly all the planes pass through (0,0,0).

$\therefore$ Their line of intersection also pass through (0, 0, 0)
Let a, b, c, be the direction ratios of required line, then we should have

$ka+4b+c=0$

$4a+kb+2c=0$

$2a+2b+c=0$

For the required line to exist the above system of equations in *a, b, c*, should have non trivial solution i.e.

$$\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix}=0$$

$\Rightarrow k(k-4)-4(4-4)+1(8-2k)=0$

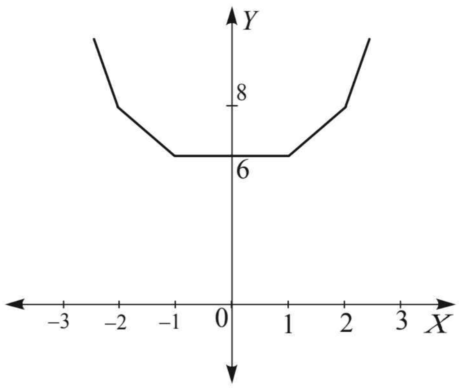
$\Rightarrow k^2-6k+8=0 \Rightarrow (k-2)(k-4)=0$

$\Rightarrow k=2$ or 4

(C) We have $f(x)=|x-1|+|x-2|+|x+1|+|x+2|$

$$=\begin{cases} -4x, & x\le -2 \\ -2x+4, & -2<x\le -1 \\ 6, & -1<x\le 1 \\ 2x+4, & 1<x\le 2 \\ 4x, & x\ge 2 \end{cases}$$

The graph of the above function is as given below



Clearly, from graph, $f(x)\ge 6$

$\Rightarrow 4k\ge 6 \Rightarrow k\ge\frac{3}{2}$

$\therefore k=2,3,4,5,6,.....$

(D) Given that

$\frac{dy}{dx}=y+1$ and $y(0)=1$

$\Rightarrow \int\frac{dy}{y+1}=\int dx \Rightarrow \ln|y+1|=x+c$

At $x=0, y=1 \Rightarrow c=\ln 2$

$\therefore \ln|y+1|=x+\ln 2 \Rightarrow y+1=2e^x \Rightarrow y=2e^x-1$

$\therefore y(\ln 2)=2e^{\ln 2}-1=2\times 2-1=3$

5. $\mathbf{(A)\to t;\ (B)\to p,r;\ (C)\to q,s;\ (D)\to r}$

Let the line through origin be $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ —(1)

then as it intersects

$\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ —(2)

and $\frac{x-8/3}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ —(3)

at P and Q, shortest distance of (1) with (2) and (3) should be zero.

$\therefore$ Using $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

we get $\begin{vmatrix} 2 & 1 & -1 \\ a & b & c \\ 1 & -2 & 1 \end{vmatrix} = 0 \Rightarrow a+3b+5c=0$ —(4)

and $\begin{vmatrix} 8/3 & -3 & 1 \\ a & b & c \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow 3a+b-5c=0$ —(5)

Solving (4) and (5), we get

$$\frac{a}{-15-5} = \frac{b}{15+5} = \frac{c}{1-9} \text{ or } \frac{a}{5} = \frac{b}{-5} = \frac{c}{2}$$

Hence equation (1) becomes $\frac{x}{5} = \frac{y}{-5} = \frac{z}{2} = \lambda$

For some value of λ, $P(5\lambda, -5\lambda, 2\lambda)$ which lies on (2) also

$$\therefore \frac{5\lambda-2}{1} = \frac{-5\lambda-1}{-2} = \frac{2\lambda+1}{1} \Rightarrow \lambda = 1$$

$\therefore$ P $(5, -5, 2)$

Also for some value of λ, $Q(5\lambda, -5\lambda, 2\lambda)$ which lies on (3) also

$$\therefore \frac{5\lambda-8/3}{3} = \frac{-5\lambda+3}{-1} = \frac{2\lambda-1}{1} \Rightarrow \lambda = 2/3$$

$$\therefore Q\left(\frac{10}{3}, \frac{-10}{3}, \frac{4}{3}\right)$$

Hence $d^2 = PQ^2 = \left(\frac{25}{9} + \frac{25}{9} + \frac{4}{9}\right) = 6$

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \tan^{-1}\frac{3}{4}$

$$\Rightarrow \tan^{-1}\left(\frac{x+3-x+3}{1+x^2-9}\right) = \tan^{-1}\left(\frac{3}{4}\right), x^2-9 \geq -1$$

$$\Rightarrow \frac{6}{x^2-8} = \frac{3}{4} \Rightarrow x^2 = 16 \text{ or } x = 4, -4$$

(C) We have $\vec{c} = \frac{\vec{a}-\mu\vec{b}}{4}$

Then $(\vec{b}-\vec{a}).(\vec{b}+\vec{c}) = 0$

$$\Rightarrow (\vec{b}-\vec{a}).\left(\vec{b}+\frac{\vec{a}-\mu\vec{b}}{4}\right) = 0$$

$$\Rightarrow (\vec{b}-\vec{a}).\left(\frac{4-\mu}{4}\vec{b}+\frac{\vec{a}}{4}\right) = 0 \Rightarrow \frac{4-\mu}{4}b^2 - \frac{a^2}{4} = 0$$

$$\Rightarrow (4-\mu)b^2 - a^2 = 0 \quad \text{—(1)}$$

Also, $2^2\left|\frac{4-\mu}{4}\vec{b}+\frac{\vec{a}}{4}\right|^2 = |\vec{b}-\vec{a}|^2$

$$\Rightarrow (4-\mu)^2b^2 + a^2 = 4b^2 + 4a^2$$

$$\Rightarrow \left[(4-\mu)^2 - 4\right]b^2 = 3a^2 \quad \text{—(2)}$$

From (1) and (2), we get

$$\frac{(4-\mu)^2-4}{4-\mu} = \frac{3}{1}$$

$$\Rightarrow \mu^2 - 8\mu + 12 = 12 - 3\mu \Rightarrow \mu^2 - 5\mu = 0$$

$$\Rightarrow \mu = 0 \text{ or } 5$$

(D) $I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin(9x/2)}{\sin(x/2)}dx = \frac{2}{\pi} \times 2\int_0^{\pi} \frac{\sin 9x/2}{\sin x/2}dx$

Let $\frac{x}{2} = 0 \Rightarrow dx = 2d\theta$

Also at $x = 0, \theta = 0$ and at $x = x, \theta = \pi/2$

$$\therefore I = \frac{8}{\pi}\int_0^{\pi/2} \frac{\sin 9\theta}{\sin\theta}d\theta$$

$$I = \frac{8}{\pi}\int_0^{\pi/2}\left[\frac{\sin 9\theta - \sin 7\theta}{\sin\theta} + \frac{(\sin 7\theta - \sin 5\theta)}{\sin\theta} + \frac{(\sin 5\theta - \sin 3\theta)}{\sin\theta} + \frac{(\sin 3\theta - \sin\theta)}{\sin\theta} + \frac{\sin\theta}{\sin\theta}\right]d\theta$$

$$= \frac{16}{\pi}\int_0^{\pi/2}(\cos 8\theta + \cos 6\theta + \cos 4\theta + \cos 2\theta + 1)d\theta + \frac{8}{\pi}\int_0^{\pi/2} d\theta$$

$$= \frac{16}{\pi}\left[\frac{\sin 8\theta}{8} + \frac{\sin 6\theta}{6} + \frac{\sin 4\theta}{4} + \frac{\sin 2\theta}{2} + \theta\right]_0^{\pi/2} + \frac{8}{\pi}(\theta)_0^{\pi/2}$$

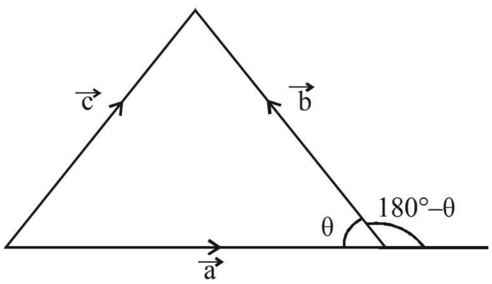
$$= 0 + \frac{8}{\pi}\left(\frac{\pi}{2} - 0\right) = 4$$

6. A→q, B→p, C→s, D→t

As $\vec{a} + \vec{b} = \vec{c}$

$\therefore$ The figure is as shown.

Clearly $\cos(180 - \theta) = \frac{\vec{a}.\vec{b}}{|a||b|} = \frac{1}{2}$



$\Rightarrow \cos\theta = -\dfrac{1}{2} \Rightarrow \theta = \dfrac{2\pi}{3}$

$\Rightarrow$ **A** $\rightarrow$ **q**

$$\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$$

$$\Rightarrow \int_a^b f(x)\,dx + \frac{3}{2}[-b^2 + a^2] = a^2 - b^2$$

$$\Rightarrow \int_a^b f(x)\,dx = -\frac{1}{2}(a^2 - b^2)$$

$$\Rightarrow \frac{d}{db}\left[\int_a^b f(x)\,dx\right] = b \Rightarrow f(b) = b \Rightarrow f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$$

$\Rightarrow$ **B** $\rightarrow$ **p**

$$\frac{\pi^2}{\ell n3}\int_{\frac{7}{6}}^{\frac{5}{6}} \sec(\pi x)dx = \frac{\pi^2}{\pi \ell n3}\left[\ell n|\sec \pi x + \tan \pi x|\right]_{\frac{7}{6}}^{\frac{5}{6}}$$

$$= \frac{\pi}{\ell n3}\left[\ell n\left|\sec\frac{5\pi}{6} + \tan\frac{5\pi}{6}\right| - \ell n\left|\sec\frac{7\pi}{6} + \tan\frac{7\pi}{6}\right|\right]$$

$$= \frac{\pi}{\ell n3}\left[\ell n\left|-\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right| - \ell n\left|-\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right|\right] = \frac{\pi}{\ell n3}\ell n3 = \pi$$

$\therefore$ **C** $\rightarrow$ **s**

For $|z| = 1$ and $z \neq 1$. Let $z = e^{i\theta}$

Then $1 - z = 1 - \cos\theta - i\sin\theta = 2\sin^2\dfrac{\theta}{2} - 2i\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

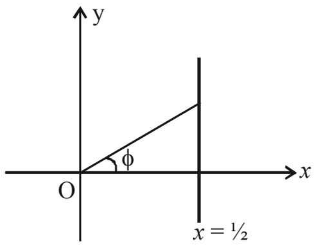
or $1 - z = 2\sin\dfrac{\theta}{2}\left[\sin\dfrac{\theta}{2} - i\cos\dfrac{\theta}{2}\right]$

$$\therefore \frac{1}{1-z} = \frac{1}{2}\left[1 + i\cot\frac{\theta}{2}\right]$$

Here real part of $\dfrac{1}{1-z}$ is always $\dfrac{1}{2}$

$\therefore$ Locus of $\dfrac{1}{1-z}$ is $x = \dfrac{1}{2}$

For which max $\left|Arg\left(\dfrac{1}{1-z}\right)\right|$ is max. value of ϕ i.e. $\dfrac{\pi}{2}$.



Clearly max. $\left|Arg\left(\dfrac{1}{1-z}\right)\right|$ approaches to $\dfrac{\pi}{2}$ but will not be attained.

$\therefore$ **D** $\rightarrow$ **t.**

7. **(c)** (P) $[\vec{a}\ \vec{b}\ \vec{c}] = 2$

$\therefore \left[2(\vec{a}\times\vec{b})\ \ 3(\vec{b}\times\vec{c})\ \ \vec{c}\times\vec{a}\right]$

$= 6\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]$

$= 6\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2 = 6\times 4 = 24$

$\therefore$ (P) $\rightarrow$ (3)

(Q) $[\vec{a}\ \vec{b}\ \vec{c}] = 5$

$\therefore \left[3(\vec{a}+\vec{b})\ \ \vec{b}+\vec{c}\ \ 2(\vec{c}+\vec{a})\right]$

$= 6\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right]$

$= 6\times 2\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 6\times 2\times 5 = 60$

$\therefore$ (Q) $\rightarrow$ (4)

(R) $\dfrac{1}{2}|\vec{a}\times\vec{b}| = 20 \Rightarrow |\vec{a}\times\vec{b}| = 40$

$\therefore \dfrac{1}{2}\left|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})\right| = \dfrac{1}{2}\left|-2\vec{a}\times\vec{b} + 3\vec{b}\times\vec{a}\right|$

$= \dfrac{1}{2}\times 5|\vec{a}\times\vec{b}| = \dfrac{5}{2}\times 40 = 100$

$\therefore$ (R) $\rightarrow$ (1)

(S) $|\vec{a}\times\vec{b}| = 30$

$\therefore \left|(\vec{a}+\vec{b})\times\vec{a}\right| = \left|(\vec{b}\times\vec{a})\right| = 30$

$\therefore$ (s) $\rightarrow$ (2)

8. **(a)** Any point on L_1 is $(2\lambda+1, -\lambda, \lambda-3)$
and that on L_2 is $(\mu+4, \mu-3, 2\mu-3)$
For point of intersection of L_1 and L_2
$2\lambda+1 = \mu+4, -\lambda = \mu-3, \lambda-3 = 2\mu-3$
$\Rightarrow$ $\lambda = 2, \mu = 1$
$\therefore$ Intersection point of L_1 and L_2 is $(5, -2, -1)$
$\because$ $ax + by + cz = d$ is perpendicular to p_1 & p_2
$\therefore$ $7a + b + 2c = 0$ and $3a + 5b - 6c = 0$

$\Rightarrow \dfrac{a}{-16} = \dfrac{b}{48} = \dfrac{c}{32} \Rightarrow \dfrac{a}{1} = \dfrac{b}{-3} = \dfrac{c}{-2}$

$\therefore$ Equation of plane is $x - 3y - 2z = d$
As it passes through $(5, -2, -1)$
$\therefore$ $5 + 6 + 2 = d = 13$
$\therefore$ $a = 1, b = -3, c = -2, d = 13$
or (P) $\rightarrow$ (3) (Q) $\rightarrow$ (2) (R) $\rightarrow$ (4) (S) $\rightarrow$ (1)

9. **(a)** $P(4)\ y = \cos(3\cos^{-1}x)$

$y = \cos\left[\cos^{-1}(4x^3 - 3x)\right]$

$y = 4x^3 - 3x$

$\Rightarrow \dfrac{dy}{dx} = 12x^2 - 3$ and $\dfrac{d^2y}{dx^2} = 24x$

$$\therefore \frac{1}{y}\left\{(x^2-1)\frac{d^2y}{dx} + x\frac{dy}{dx}\right\}$$

$$= \frac{1}{4x^3 - 3x}\left\{(x^2-1)24x + x(12x^2 - 3)\right\}$$

$= \frac{3x}{4x^3-3x}\{8x^2-8+4x^2-1\}$

$= \frac{3x\{12x^2-9\}}{4x^3-3x} = \frac{9\{4x^3-3x\}}{4x^3-3x} = 9$

$Q(3)$ $\because A_1, A_2, A_3, \ldots A_n$ are the vertices of a regular polygon of n sides with its centre at origin and $\vec{a}_1, \vec{a}_2$, ... $\vec{a}_n$ are their position vectors.

$\therefore \quad |\vec{a}_1| = |\vec{a}_2| = \ldots = |\vec{a}_n| = \lambda$

Then $\vec{a}_k \times \vec{a}_{k+1} = \lambda^2 \sin\frac{2\pi}{n}$

and $\vec{a}_k \times \vec{a}_{k+1} = \lambda^2 \cos\frac{2\pi}{n}$

Hence given equation reduces to

$(n-1)\lambda^2 \sin\frac{2\pi}{n} = (n-1)\lambda^2 \cos\frac{2\pi}{n}$

$\Rightarrow \tan\frac{2\pi}{n} = 1 \Rightarrow \frac{2\pi}{n} = \frac{\pi}{4} \Rightarrow n = 8$

$R.(2)$ Normal from $P(h, 1)$ on $\frac{x^2}{6}+\frac{y^2}{3}=1$ is

$\frac{x-h}{h/6} = \frac{y-1}{1/3}$

$\Rightarrow 2(x-h) = h(y-1)$

$\Rightarrow 2x - hy - h = 0$

It is perpendicular to $x + y = 8$

$\therefore \frac{2}{h}\times -1 = -1 \Rightarrow h = 2$

$S.(1)$ $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{2x+1}\cdot\frac{1}{4x+1}}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{4x+1+2x+1}{8x^2+6x+1-1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{6x+2}{8x^2+6x}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \tan^{-1}\left(\frac{3x+1}{4x^2+3x}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow \frac{3x+1}{4x^2+3x} = \frac{2}{x^2}$

$\Rightarrow 3x^2 - 7x - 6 = 0$ (for $x > 0$)

$\Rightarrow x = 3$ or $-\frac{2}{3}$ (rejected as $x > 0$)

$\therefore$ Only one +ve solution is there

Hence (a) is the correct option.

10. (A) $\to$ q; (B) $\to$ p, q; (C) $\to$ p, q, s, t; (D) $\to$ q, t

(A) $\frac{\sqrt{3}\alpha+\beta}{2} = \sqrt{3} \Rightarrow \alpha = \frac{2\sqrt{3}-\beta}{\sqrt{3}}$

$\therefore \quad \frac{2\sqrt{3}-\beta}{\sqrt{3}} = 2+\sqrt{3}\beta \Rightarrow \beta = 0 \Rightarrow \alpha = 2$

(B) $Lf'(1) = -6a$ and $Rf'(1) = b$

$-6a = b$ ...(i)

Also f is continuous at $x = 1$,

$\therefore \quad -3a - 2 = b + a^2$

$\Rightarrow \quad a^2 - 3a + 2 = 0$ (using (i))

$\Rightarrow \quad a = 1, 2$

(C) $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$

$\Rightarrow \quad (3 - 3\omega + 2\omega^2)^{4n+3} + \left(\frac{2\omega^2+3-3\omega}{\omega^2}\right)^{4n+3} + \left(\frac{-3\omega+2\omega^2+3}{\omega}\right)^{4n+3} = 0$

$\Rightarrow \quad (3-3\omega+2\omega^2)^{4n+3}[1+\omega^{4n+3}+(\omega^2)^{4n+3}] = 0$

$\Rightarrow \quad 4n+3$ should be an integer other than multiple of 3.

$\therefore \quad n = 1, 2, 4, 5$

(D) $\frac{2ab}{a+b} = 4 \Rightarrow ab = 2a + 2b$ ...(i)

Also $a + q = 10$ or $a = 10 - q$

and $b + 5 = 2q$ or $b = 2q - 5$

Putting values of a and b in eq^n(i)

$q = 4$ or $\frac{15}{2} \Rightarrow a = 6$ or $\frac{5}{2}$

$\therefore \quad |q - a| = 2$ or 5.

11. (A)$\to$p, r, s; (B) $\to$ p; (C) $\to$ p, q; (D)$\to$s, t

(A) $2(a^2-b^2) = c^2$

$\Rightarrow \quad 2(\sin^2 x - \sin^2 y) = \sin^2 z$

$\Rightarrow \quad 2\sin(x+y)\sin(x-y) = \sin^2 z$

$\Rightarrow \quad 2\sin(x-y) = \sin z$ ($\because \sin(x+y) = \sin z$)

$\Rightarrow \quad \frac{\sin(x-y)}{\sin z} = \frac{1}{2} = \lambda$

$\therefore \quad \cos(n\pi\lambda) = 0 \Rightarrow \cos\frac{n\pi}{2} = 0 \Rightarrow n = 1, 3, 5$

(B) $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$

$\Rightarrow \quad 2\cos^2 X - 2\cos 2Y = 2\sin X \sin Y$

$\Rightarrow \quad 1 - \sin^2 X - 1 + 2\sin^2 Y = \sin X \sin Y$

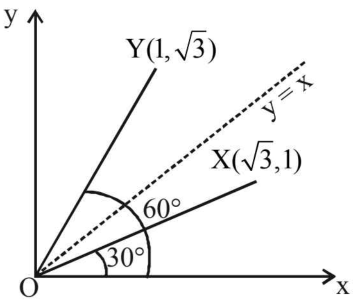
$\Rightarrow \quad \sin^2 X + \sin X \sin Y - 2\sin^2 Y = 0$

$\Rightarrow \quad (\sin X - \sin Y)(\sin X + 2\sin Y) = 0$

$\Rightarrow \quad \dfrac{\sin X}{\sin Y} = 1 \text{ or } -2$

$\therefore \quad \dfrac{a}{b} = 1.$

(C) $X(\sqrt{3}, 1), Y(1, \sqrt{3}), Z(\beta, 1-\beta)$



By symmetry, acute angle bisector of $\angle XOY$ is $y = x$.

$\therefore$ Distance of Z from bisector

$= \left|\dfrac{\beta - 1 + \beta}{\sqrt{2}}\right| = \dfrac{3}{\sqrt{2}} \Rightarrow 2\beta - 1 = \pm 3 \text{ or } \beta = 2 \text{ or } -1$

$\therefore \quad |\beta| = 1, 2$

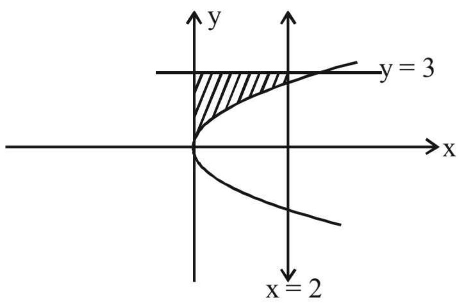
(D) For $\alpha = 0, y = 3$

For $\alpha = 1, y = |x-1| + |x-2| + x$

Case I

F(α) is the area bounded by $x = 0, x = 2, y^2 = 4x$ and $y = 3$

$\therefore \quad F(\alpha) = \int_0^2 (3 - 2\sqrt{x})dx$



$= \left|3x - \dfrac{4x\sqrt{x}}{3}\right|_0^2 = 6 - \dfrac{8\sqrt{2}}{3}$

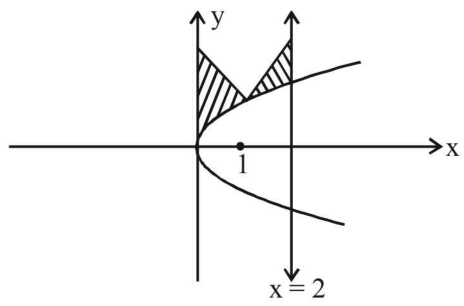
$\therefore \quad F(\alpha) + \dfrac{8}{3}\sqrt{2} = 6$

Case II

F(α) is the area bounded by $x = 0$, $x = 2$, $y^2 = 4x$ and $y = |x-1| + |x-2| + x$

$= \begin{cases} 3 - x, 0 \le x < 1 \\ x + 1, 1 \le x \le 2 \end{cases}$

$\therefore \quad F(\alpha) = \int_0^1 (3 - x - 2\sqrt{x})dx + \int_1^2 (x + 1 - 2\sqrt{x})dx$



$= \left(3x - \dfrac{x^2}{2} - \dfrac{4x}{3}\sqrt{x}\right)_0^1 + \left(\dfrac{x^2}{2} + x - \dfrac{4}{3}x\sqrt{x}\right)_1^2$

$= 3 - \dfrac{1}{2} - \dfrac{4}{3} + 2 + 2 - \dfrac{8\sqrt{2}}{3} - \dfrac{1}{2} - 1 + \dfrac{4}{3} = 5 - \dfrac{8\sqrt{2}}{3}$

$F(\alpha) + \dfrac{8\sqrt{2}}{3} = 5$

G. Comprehension Based Questions

1. (b) Vector in the direction of $L_1 = \vec{n_1} = 3\hat{i} + \hat{j} + 2\hat{k}$

Vector in the direction of $L_2 = \vec{n_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ Vector perpendicular to both L_1 and L_2

$= \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$

$\therefore$ Required unit vector

$= \hat{n} = \dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{1 + 49 + 25}} = \dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

2. (d) The shortest distance between L_1 and L_2 is

$= \dfrac{(\vec{a}_2 - \vec{a}_1).\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} = (\vec{a}_2 - \vec{a}_1).\hat{n}$

where $a_1 = -\hat{i} - 2\hat{j} - \hat{k} \quad a_2 = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 4\hat{k} \qquad \therefore (\vec{a}_2 - \vec{a}_1).\hat{n}$

$\therefore \quad (3\hat{i} + 4\hat{k}).\left(\dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}\right) = \dfrac{-3 + 20}{5\sqrt{3}} = \dfrac{17}{5\sqrt{3}}$

3. (c) The plane passing through (–1, –2, –1) and having normal along $\vec{n}$ is

$-1(x+1) - 7(y+2) + 5(z+1) = 0$

or $x + 7y - 5z + 10 = 0$

$\therefore$ Distance of point (1, 1, 1) from the above plane is

$= \dfrac{1 + 7 \times 1 - 5 \times 1 + 10}{\sqrt{1 + 49 + 25}} = \dfrac{13}{\sqrt{75}}$

H. Assertion & Reason Type Questions

1. (d) The line of intersection of given plane is

$3x - 6y - 2z - 15 = 0 = 2x + y - 2z - 5$

For $z = 0$, we obtain $x = 3$ and $y = -1$

$\therefore$ Line passes through $(3, -1, 0)$.

Let a, b, c be the d'rs of line of intersection, then

$3a - 6b - 2c = 0$ and $2a + b - 2c = 0$

Solving the above equations using cross product method, we get $a:b:c = 14:2:15$

$\therefore$ Eqn. of line is $\dfrac{x-3}{14} = \dfrac{y+1}{2} = \dfrac{z}{15} = t$

whose parametric form is

$x = 3+14t, y = 1+2t, z = 15t$

$\therefore$ Statement-I is false (value of y is not matching).

Since dr's of line intersection of given planes are 14, 2, 15

$\therefore$ $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to this line.

$\therefore$ Statement 2 is true.

2. **(c)** $\overrightarrow{PQ}\times(\overrightarrow{RS}+\overrightarrow{ST}) = \overrightarrow{PQ}\times\overrightarrow{RT}$

$= |\overrightarrow{PQ}|\times|\overrightarrow{RT}|\sin 150\hat{n} \neq 0$

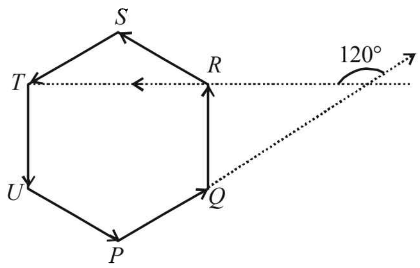
$\neq 0$

Statement-1 is true.

Also, $\overrightarrow{PQ}\times\overrightarrow{RS} = |\overrightarrow{PQ}|\times|\overrightarrow{RS}|\sin 120°\times\hat{n}_1 \neq 0$

And $\overrightarrow{PQ}\times\overrightarrow{ST} = \overrightarrow{PQ}|\times|\overrightarrow{ST}|\sin 180°\times\hat{n}_2 = 0$

$\therefore$ Statement-2 is false.



3. **(d)** The given planes are

$P_1 : x-y+z=1$ ...(1)

$P_2 : x+y-z=-1$ ...(2)

$P_3 : x-3y+3z=2$ ...(3)

Line L_1 is intersection of planes P_2 and P_3.

$\therefore L_1$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -3 & 3 \end{vmatrix} = -4\hat{j}-4\hat{k}$$

Line L_2 is intersection of P_3 and P_1

$\therefore L_2$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -3 & 3 \end{vmatrix} = -2\hat{j}-2\hat{k}$$

Line L_3 is intersection of P_1 and P_2

$\therefore L_3$ is parallel to the vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 2\hat{j}+2\hat{k}$$

Clearly lines L_1, L_2 and L_3 are parallel to each other.

$\therefore$ Statement–1 is False

Also family of planes passing through the intersection of P_1 and P_2 is $P_1+\lambda P_2 = 0$. If plane P_3 is represented by $P_1+\lambda P_2 = 0$ for some value of λ then the three planes pass through the same point.

Here $P_1+\lambda P_2 = 0$

$\Rightarrow x(1+\lambda)+y(\lambda-1)+z(1-\lambda)+(\lambda-1) = 0$

This will be identical to P_3 if

$$\frac{1+\lambda}{1} = \frac{\lambda-1}{-3} = \frac{1-\lambda}{3} = \frac{1-\lambda}{2} \quad ...(1)$$

Taking $\dfrac{1+\lambda}{1} = \dfrac{1-\lambda}{2}$, we get $\lambda = -\dfrac{1}{3}$ and taking $\dfrac{1+\lambda}{1} = \dfrac{1-\lambda}{3}$, we get $\lambda = -\dfrac{2}{3}$.

$\therefore$ There is no value of λ which satisfies eq (1).

$\therefore$ The three planes do not have a common point.

$\Rightarrow$ Statement 2 is true.

$\therefore$ (d) is the correct option.

I. Integer Value Correct Type

1. **(5)** We have $\vec{a} = \dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}, \vec{b} = \dfrac{2\hat{i}+\hat{j}+3\hat{k}}{\sqrt{14}}$

Clearly $|\vec{a}| = 1$ $|\vec{b}| = 1$ and $\vec{a}.\vec{b} = 0$

$(2\vec{a}+\vec{b}).\left[(\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b})\right]$

$= -(2\vec{a}+\vec{b}).\left[(\vec{a}-2\vec{b})\times(\vec{a}\times\vec{b})\right]$

$= -(2\vec{a}+\vec{b}).\left[((\vec{a}-2\vec{b}).\vec{b})\vec{a}-((\vec{a}-2\vec{b}).\vec{a})\vec{b}\right]$

$= -(2\vec{a}+\vec{b}).\left[\left(\vec{a}.\vec{b}-2|\vec{b}|^2\right)\vec{a}-\left(|\vec{a}|^2-2\vec{b}.\vec{a}\right)\vec{b}\right]$

$= -(2\vec{a}+\vec{b}).\left[-2\vec{a}-\vec{b}\right]$

$= (2\vec{a}+\vec{b}).(2\vec{a}+\vec{b}) = 4|\vec{a}|^2+|\vec{b}|^2 \quad (\because \vec{a}.\vec{b} = 0)$

$= 4+1 = 5.$

2. **(6)** The equation of plane containing the lines

$\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$ is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0 \Rightarrow x-2y+z = 0$$

$\therefore$ Distance between $x-2y+z=0$ and $Ax-2y+z=d$

= Perpendicular distance between parallel planes $(\therefore A = 1)$

$= \dfrac{|d|}{\sqrt{6}} = \sqrt{6} \Rightarrow |d| = 6.$

3. **(9)** We have $\vec{r}\times\vec{b}-\vec{c}\times\vec{b}=\vec{0}$

$\Rightarrow (\vec{r}-\vec{c})\times\vec{b}=\vec{0} \Rightarrow \vec{r}-\vec{c} \parallel \vec{b}$

Let $\vec{r}-\vec{c}=\lambda\vec{b}$ or $\vec{r}=\vec{c}+\lambda\vec{b}$

$\Rightarrow \vec{r}=\hat{i}+2\hat{j}+3\hat{k}-\lambda\hat{i}+\lambda\hat{j}=(1-\lambda)i+(2+\lambda)\hat{j}+3\hat{k}$

$\vec{r}.\vec{a}=0 \Rightarrow -1+\lambda-3=0 \Rightarrow \lambda=4$

$\therefore\ \vec{r}=-3\hat{i}+6\hat{j}+3\hat{k}$

$\therefore\ \vec{r}.\vec{b}=3+6=9$

4. **(3)** $\vec{a},\vec{b},\vec{c}$ are units vectors such that

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$

$\Rightarrow 2\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)-2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=9$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\frac{-3}{2}$

Also $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$

$=1+1+1+2\times\left(-\frac{3}{2}\right)=0$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0 \Rightarrow (\vec{b}+\vec{c})=-\vec{a}$

$\therefore\ |2\vec{a}+5(\vec{b}+\vec{c})|=|2\vec{a}-5\vec{a}|=|-3\vec{a}|=3$

5. **(5)** Given 8 vectors are
(1, 1, 1), (–1, –1, –1); (–1, 1, 1), (1, –1, –1); (1, –1, 1), (–1, 1, –1); (1, 1, –1), (–1, –1, 1)
These are 4 diagonals of a cube and their opposites.
For 3 non coplanar vectors first we select 3 groups of diagonals and its opposite in ${}^4C_3$ ways. Then one vector from each group can be selected in $2\times2\times2$ ways.

$\therefore$ Total ways $={}^4C_3\times2\times2\times2=32=2^5$

$\therefore\ p=5$

6. **(5)** Let k, $k+1$ be removed from pack.

$\therefore (1+2+3+...+n)-(k+k+1)=1224$

$\frac{n(n+1)}{2}-2k=1225 \Rightarrow k=\frac{n(n+1)-2450}{4}$

for $n=50$, $k=25$ $\therefore$ $k-20=5$

7. **(4)** $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=\cos\frac{\pi}{3}=\frac{1}{2}$

Given $p\vec{a}+q\vec{b}+r\vec{c}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}$

Taking its dot product with $\vec{a},\vec{b},\vec{c}$, we get

$$p+\frac{1}{2}q+\frac{1}{2}r=\left[\vec{a}\ \vec{b}\ \vec{c}\right] \quad ...(1)$$

$$\frac{1}{2}p+q+\frac{1}{2}r=0 \quad ...(2)$$

$$\frac{1}{2}p+\frac{1}{2}q+r=\left[\vec{a}\ \vec{b}\ \vec{c}\right] \quad ...(3)$$

From (1) and (3), $p=r$ Using (2) $q=-p$

$\therefore\ \frac{p^2+2q^2+r^2}{q^2}=\frac{p^2+2p^2+p^2}{p^2}=4$

8. **(9)** $\vec{s}=4\vec{p}+3\vec{q}+5\vec{r}$

$\vec{s}=x(-\vec{p}+\vec{q}+\vec{r})+y(\vec{p}-\vec{q}+\vec{r})+z(-\vec{p}-\vec{q}+\vec{r})$

$\Rightarrow\ -x+y-z=4$

$x-y-z=3$

$x+y+z=5$

Solving above equations $x=4$, $y=\frac{9}{2}$, $z=\frac{-7}{2}$

$\therefore\ 2x+y+z=9$

Section-B JEE Main/ AIEEE

1. (a) As the point (3, 2, 0) lies on the given line $\frac{x-4}{1}=\frac{y-7}{5}=\frac{z-4}{4}$

$\therefore$ There can be infinite many planes passing through this line. But here out of the four options only first option is satisfied by the coordinates of both the points (3, 2, 0) and (4, 7, 4)

$\therefore$ $x-y+z=1$ is the required plane.

2. (b) $\left(\vec{a}\times\vec{b}\right)^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\frac{\pi}{6} = 16\times4\times\frac{1}{4}=16$

3. (a) We have, $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]$

$=(\vec{a}\times\vec{b}).\left\{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\right\}$

$=(\vec{a}\times\vec{b}).\left\{(\vec{m}\cdot\vec{a})\vec{c}-(\vec{m}\cdot\vec{c})\vec{a}\right\}$

(where $\vec{m}=\vec{b}\times\vec{c}$)

$=\{(\vec{a}\times\vec{b}).\vec{c}\}.\{(\vec{a}\cdot(\vec{b}\times\vec{c})\}=[\vec{a}\ \vec{b}\ \vec{c}]^2=4^2=16.$

4. (a) $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow (\vec{b}+\vec{c})^2=(\vec{a})^2=5^2+3^2+2\vec{b}\cdot\vec{c}=7^2$

$\Rightarrow 2|\vec{b}||\vec{c}|\cos\theta=49-34=15; \Rightarrow 2\times5\times3\cos\theta=15;$

$\Rightarrow \cos\theta=1/2; \Rightarrow \theta=\frac{\pi}{3}=60°$

5. (a) We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow(\vec{a}+\vec{b}+\vec{c})^2=0$

$\Rightarrow|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow 25+16+9+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=-25.$

$\therefore|\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}|=25.$

6. (a) Since $\vec{a},\vec{c},\vec{b}$ form a right handed system,

$\therefore\quad \vec{c}=\vec{b}\times\vec{a}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&0\\x&y&z\end{vmatrix}=z\hat{i}-x\hat{k}$

7. (b) We have $\vec{a}\times\vec{b}=39\vec{k}=\vec{c}$

Also $|\vec{a}|=\sqrt{34}, |\vec{b}|=\sqrt{45}, |\vec{c}|=39$;

$\therefore|\vec{a}|:|\vec{b}|:|\vec{c}|=\sqrt{34}:\sqrt{45}:39.$

8. (c) Let $\vec{a}+\vec{b}+\vec{c}=\vec{r}$. Then

$\vec{a}\times(\vec{a}+\vec{b}+\vec{c})=\vec{a}\times\vec{r}\Rightarrow0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times\vec{r}$

$\Rightarrow\vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{a}\times\vec{r}\Rightarrow\vec{a}\times\vec{r}=\vec{0}$

Similarly $\vec{b}\times\vec{r}=\vec{0}$ & $\vec{c}\times\vec{r}=\vec{0}$

Above three conditions will be satisfied for non-zero vectors if and only if $\vec{r}=\vec{0}$

9. (b) Equation of plane through (1, 0, 0) is

$a(x-1)+by+cz=0$ ...(i)

(i) passes through (0, 1, 0).

$-a+b=0\Rightarrow b=a$; Also, cos 45°

$=\frac{a+a}{\sqrt{2(2a^2+c^2)}}\Rightarrow 2a=\sqrt{2a^2+c^2}\Rightarrow 2a^2=c^2$

$\Rightarrow c=\sqrt{2}a$.

So d.r of normal are a, a $\sqrt{2}a$ i.e. 1, 1, $\sqrt{2}$.

10. (a) since $\vec{n}$ is perpendicular $\vec{u}$ *and* $\vec{v}$, $\vec{n}=\frac{\vec{u}\times\vec{v}}{|\vec{u}||\vec{v}|}$

$\hat{n}=\frac{\begin{vmatrix}i&j&k\\1&1&0\\1&-1&0\end{vmatrix}}{\sqrt{2}\times\sqrt{2}}=\frac{-2\hat{k}}{2}=-\hat{k}$

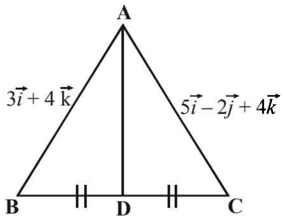
$|\vec{\omega}.\hat{n}|=|(i+2j+3k).(-k)|=|-3|=3$

11. (d) $\vec{F}+\vec{F}_1+\vec{F}_2=7i+2j-4k$

$\vec{d}=P.V\ of\ \vec{B}-P.V\ of\ \vec{A}=4i+2j-2k$

$W=\vec{F}.\vec{d}=28+4+8=40$ *unit*

12. (d)



$P.V\ of\ \overline{AD}=\frac{(3+5)i+(0-2)j+(4+4)k}{2}$

$=4i-j+4k\ or\ |\overline{AD}|=\sqrt{16+16+1}=\sqrt{33}$

13. (d) Shortest distance = perpendicular distance between the plane and sphere = distance of plane from centre of sphere – radius

$=\left|\frac{-2\times12+4\times1+3\times3-327}{\sqrt{144+9+16}}\right|-\sqrt{4+1+9+155}$

$=26-13=13$

14. (a) $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c};\ \frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'}.$

For perpendicularity of lines $aa'+1+cc'=0$

15. (d) $\begin{vmatrix}x_2-x_1&y_2-y_1&z_2-z_1\\l_1&m_1&n_1\\l_2&m_2&n_2\end{vmatrix}=0$

$$\therefore \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 0 & -1 \\ 2 & 1+k & -k \\ k+2 & 1 & 1 \end{vmatrix} = 0$$

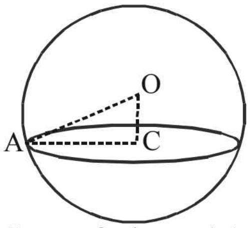
$$k^2 + 3k = 0 \Rightarrow k(k+3) = 0 \text{ or } k = 0 \text{ or } -3$$

16. **(c)** $\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow (\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})=0$

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$$

$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=\frac{-1-4-9}{2}=-7$$

17. **(d)**



Centre of sphere = (–1, 1, 2)

Radius of sphere $\sqrt{1+1+4+19}=5$

Perpendicular distance from centre to the plane

$$OC = d = \left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = \frac{12}{3} = 4.$$

$$AC^2 = AO^2 - OC^2 = 5^2 - 4^2 = 9 \Rightarrow AC = 3$$

18. **(b)** Vector perpendicular to the face OAB

$$= \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

Vector perpendicular to the face ABC

$$= \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{i} - 5\hat{j} - 3\hat{k}$$

Angle between the faces = angle between their normals

$$\cos\theta = \left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right| = \frac{19}{35} \text{ or } \theta = \cos^{-1}\left(\frac{19}{35}\right)$$

19. **(c)**
$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$As \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \text{ (given condition)} \quad \therefore abc = -1$$

20. $A=(7,-4,7), B=(1,-6,10), C=(-1,-3,4)$ and $D=(5,-1,5)$

$$AB = \sqrt{(7-1)^2+(-4+6)^2+(7-10)^2}$$
$$= \sqrt{36+4+9} = 7$$

Similarly $BC = 7, CD = \sqrt{41}, DA = \sqrt{17}$

$\therefore$ None of the options is satisfied

21. **(c)** $(\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}-\vec{v}\times\vec{v}+\vec{v}\times\vec{w})$

$$= (\vec{u}+\vec{v}-\vec{w}).(\vec{u}\times\vec{v}-\vec{u}\times\vec{w}+\vec{v}\times\vec{w}) = \vec{u}.(\vec{u}\times\vec{v})$$
$$-\vec{u}.(\vec{u}\times\vec{w})+\vec{u}.(\vec{v}\times\vec{w})+\vec{v}.(\vec{u}\times\vec{v})-\vec{v}.(\vec{u}\times\vec{w})$$
$$+\vec{v}.(\vec{v}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})+\vec{w}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{w})$$
$$= \vec{u}.(\vec{v}\times\vec{w})-\vec{v}.(\vec{u}\times\vec{w})-\vec{w}.(\vec{u}\times\vec{v})$$
$$= [\vec{u}\vec{v}\vec{w}]+[\vec{v}\vec{w}\vec{u}]-[\vec{w}\vec{u}\vec{v}] = \vec{u}.(\vec{v}\times\vec{w})$$

22. **(a)** Eq. of planes be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \;\&\; \frac{x}{a'}+\frac{y}{b'}+\frac{z}{c'}=1$

($\perp r$ distance on plane from origin is same.)

$$\left|\frac{-1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}\right| = \left|\frac{-1}{\sqrt{\frac{1}{a'^2}+\frac{1}{b'^2}+\frac{1}{c'^2}}}\right|$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}-\frac{1}{a'^2}-\frac{1}{b'^2}-\frac{1}{c'^2}=0$$

23. **(c)** The planes are $2x+y+2z-8=0$. ...(1)

and $4x+2y+4z+5=0$

or $2x+y+2z+\frac{5}{2}=0$ ...(2)

$\therefore$ Distance between (1) and (2)

$$= \left|\frac{\frac{5}{2}+8}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{21}{2\sqrt{9}}\right| = \frac{7}{2}$$

24. **(b)** Let a point on the line $x = y+a = z$ is $(\lambda, \lambda-a, \lambda)$ and a point on the line $x+a = 2y = 2z$ is $\left(\mu-a, \frac{\mu}{2}, \frac{\mu}{2}\right)$, then direction ratio of the line joining these points are $\lambda-\mu+a,\ \lambda-a-\frac{\mu}{2},\ \lambda-\frac{\mu}{2}$

If it respresents the required line, then

$$\frac{\lambda-\mu+a}{2} = \frac{\lambda-a-\frac{\mu}{2}}{1} = \frac{\lambda-\frac{\mu}{2}}{2}$$

on solving we get $\lambda = 3a, \mu = 2a$

$\therefore$ The required points of intersection are $(3a, 3a-a, 3a)$ and $\left(2a-a, \frac{2a}{2}, \frac{2a}{2}\right)$

or $(3a, 2a, 3a)$ and (a, a, a)

25. (d) The given lines are

$$x-1=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s \text{ ....(1)}$$

and $2x=y-1=\frac{z-2}{-1}=t$ ....(2)

The lines are coplanar, if

$$\begin{vmatrix} 0-(-1) & -1-3 & -2-(-1) \\ 1 & -\lambda & \lambda \\ \frac{1}{2} & 1 & -1 \end{vmatrix}=0$$

$$c_2 \to c_2+c_3; \begin{vmatrix} 1 & -5 & -1 \\ 1 & 0 & \lambda \\ \frac{1}{2} & 0 & -1 \end{vmatrix}=0$$

$$\Rightarrow 5(-1-\frac{\lambda}{2})=0 \Rightarrow \lambda=-2$$

26. (a) The equations of spheres are

$S_1: x^2+y^2+z^2+7x-2y-z-13=0$ *and*

$S_2: x^2+y^2+z^2-3x+3y+4z-8=0$

Their plane of intersection is

$S_1-S_2=0 \Rightarrow 10x-5y-5z-5=0$

$\Rightarrow 2x-y-z=1$

27. (c) Let $\vec{a}+2\vec{b}=t\vec{c}$ and $\vec{b}+3\vec{c}=s\vec{a}$, where t and s are scalars. Adding, we get

$\vec{a}+3\vec{b}+3\vec{c}=t\vec{c}+s\vec{a} \Rightarrow \vec{a}+2\vec{b}+6\vec{c}=t\vec{c}+s\vec{a}-\vec{b}+3\vec{c}$

$=t\vec{c}+(\vec{b}+3\vec{c})-\vec{b}+3\vec{c}=(t+6)\vec{c}$

$\left[\text{using } s\,\vec{a}=\vec{b}+3\vec{c}\right]$

$=\lambda\vec{c}$, where $\lambda=t+6$

28. (d) Resultant of forces $\vec{F}=7\hat{i}+2\hat{j}-4\hat{k}$

Disptacement $\vec{d}=4\hat{i}+2\hat{j}-2\hat{k}$

$\therefore$ Work done $=\vec{F}.\vec{d}=28+4+8=40$

29. (c) Vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+4\vec{c}$, and $(2\lambda-1)\vec{c}$ are

coplanar if $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda-1 \end{vmatrix}=0$

$\Rightarrow \lambda(2\lambda-1)=0 \Rightarrow \lambda=0$ or $\frac{1}{2}$

$\therefore$ Forces are noncoplanar for all λ, except $\lambda=0, \frac{1}{2}$

30. (c) Projection of $\vec{v}$ along $\vec{u}$ $=\frac{\vec{v}.\vec{u}}{|\vec{u}|}=\frac{\vec{v}.\vec{u}}{2}$

projection of $\vec{w}$ along $\vec{u}$ $=\frac{\vec{w}.\vec{u}}{|\vec{u}|}=\frac{\vec{w}.\vec{u}}{2}$

Given $\frac{\vec{v}.\vec{u}}{2}=\frac{\vec{w}.\vec{u}}{2}$ ....(1)

Also , $\vec{v}.\vec{w}=0$ ....(2)

Now $|\vec{u}-\vec{v}+\vec{w}|^2$

$=|\vec{u}|^2+|\vec{v}|^2+|\vec{w}|^2-2\vec{u}.\vec{v}-2\vec{v}.\vec{w}+2\vec{u}.\vec{w}$

$=1+4+9+0$ [From (1) and (2)] $=14$

$\therefore |\vec{u}-\vec{v}+\vec{w}|=\sqrt{14}$

31. (a) Given $(\vec{a}\times\vec{b})\times\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Clearly $\vec{a}$ and $\vec{b}$ are noncollinear

$\Rightarrow (\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\therefore\ \vec{a}.\vec{c}=0$ and $-\vec{b}.\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}| \Rightarrow \cos\theta=\frac{-1}{3}$

$\therefore \sin\theta=\sqrt{1-\frac{1}{9}}=\frac{2\sqrt{2}}{3}$

[θ is acute angle between $\vec{b}$ and $\vec{c}$]

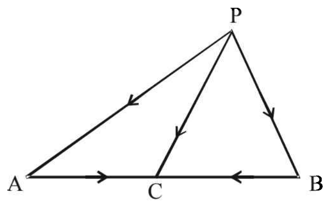
32. (a) $\overline{PA}+\overline{AP}=0$ and $\overline{PC}+\overline{CP}=0$

$\Rightarrow\ \overline{PA}+\overline{AC}+\overline{CP}=0$ and $\overline{PB}+\overline{BC}+\overline{CP}=0$

Adding, we get $\overline{PA}+\overline{PB}+\overline{AC}+\overline{BC}+2\overline{CP}=0.$

Since $\overline{AC}=-\overline{BC}$ & $\overline{CP}=-\overline{PC}$

$\Rightarrow\ \overline{PA}+\overline{PB}-2\overline{PC}=0.$



33. (a) If θ is the angle between line and plane then $\left(\frac{\pi}{2}-\theta\right)$ is the angle between line and normal to plane given by

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{(\hat{i}+2\hat{j}+2\hat{k}).(2\hat{i}-\hat{j}+\sqrt{\lambda}\hat{k})}{3\sqrt{4+1+\lambda}}$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

$$\Rightarrow \sin\theta=\frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}}=\frac{1}{3} \Rightarrow 4\lambda=5+\lambda \Rightarrow \lambda=\frac{5}{3}$$

34. (b) The given lines are $2x=3y=-z$

or $\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ [Dividing by 6]

and $6x=-y=-4z$

or $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$ [Dividing by 12]

$\therefore$ Angle between two lines is

$$\cos\theta=\frac{3.2+2.(-12)+(-6).(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$$

$$=\frac{6-24+18}{\sqrt{49}\sqrt{157}}=0\Rightarrow\theta=90^\circ$$

35. (c) Centers of given spheres are $(-3, 4, 1)$ and $(5, -2, 1)$.
Mid point of centres is $(1, 1, 1)$.
Satisfying this in the equation of plane, we get
$2a-3a+4a+6=0 \quad \Rightarrow a=-2$.

36. (b) A point on line is $(2, -2, 3)$ its perpendicular distance from the plane $x+5y+z-5=0$ is

$$=\left|\frac{2-10+3-5}{\sqrt{1+25+1}}\right|=\frac{10}{3\sqrt{3}}$$

37. (c) Let $\vec{a}=x\vec{i}+y\vec{j}+z\vec{k}$

$\vec{a}\times\vec{i}=z\vec{j}-y\vec{k} \Rightarrow(\vec{a}\times\vec{i})^2=y^2+z^2$

Similarly, $(\vec{a}\times\vec{j})^2=x^2+z^2$ and $(\vec{a}\times\vec{k})^2=x^2+y^2$

$\Rightarrow (\vec{a}\times\vec{i})^2+(\vec{a}\times\vec{j})^2+(\vec{a}\times\vec{k})^2$

$=2(x^2+y^2+z^2)=2\vec{a}^2$

38. (c) a, b, c are in H.P. $\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$\Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}\Rightarrow\frac{1}{a}-\frac{2}{b}+\frac{1}{c}=0$

$\therefore \frac{x}{a}+\frac{y}{a}+\frac{1}{c}=0$ passes through $(1, -2)$

39. (a) Vector $a\vec{i}+a\vec{j}+c\vec{k}$, $\vec{i}+\vec{k}$ and $c\vec{i}+c\vec{j}+b\vec{k}$ are coplanar

$$\begin{vmatrix} a & a & c\\ 1 & 0 & 1\\ c & c & b\end{vmatrix}=0\Rightarrow c^2=ab\Rightarrow c=\sqrt{ab}$$

$\therefore$ c is G.M. of a and b.

40. (b) $[\lambda(\vec{a}+\vec{b})\ \lambda^2\vec{b}\ \ \lambda\vec{c}]=[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$

$\Rightarrow \lambda^4[\vec{a}+\vec{b}\ \ \vec{b}\vec{c}]=[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{b}]$

$\Rightarrow\lambda^4\{[\vec{a}\vec{b}\vec{c}]+[\vec{b}\vec{b}\vec{c}]\}=[\vec{a}\ \vec{b}\ \vec{b}]+[\vec{a}\ \vec{c}\ \vec{b}]$

$\Rightarrow \lambda^4[\vec{a}\ \vec{b}\ \vec{c}]=-[\vec{a}\ \vec{b}\ \vec{c}] \Rightarrow \lambda^4=-1$

$\Rightarrow \lambda$ has no real values.

41. (d) $\vec{a}=\hat{i}-\hat{k}$, $\vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and

$\vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}]=\vec{a}.\vec{b}\times\vec{c}=\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}$$

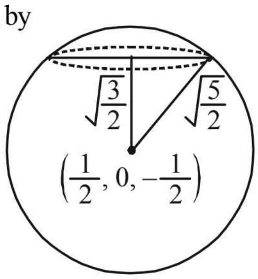
$=1\left[1+x-y-x+x^2\right]-\left[-x^2-y\right]$

$=1-y+x^2-x^2+y=1$

Hence $[\vec{a}\ \vec{b}\ \vec{c}]$ is independent of x and y both.

42. (b) Perpendicular distance of centre $\left(\frac{1}{2},0,-\frac{1}{2}\right)$ from $x+2y-2=4$ is given by

$$\frac{\left|\frac{1}{2}+\frac{1}{2}-4\right|}{\sqrt{6}}=\sqrt{\frac{3}{2}}$$



radius of sphere

$$=\sqrt{\frac{1}{4}+\frac{1}{4}+2}=\sqrt{\frac{5}{2}}$$

$\therefore$ radius of circle $=\sqrt{\frac{5}{2}-\frac{3}{2}}=1$.

43. (d) $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{a}.\vec{b}\neq0$, $\vec{b}.\vec{c}\neq0$

$\Rightarrow (\vec{a}.\vec{c}).\vec{b}-(\vec{b}.\vec{c})\vec{a}=(\vec{a}.\vec{c}).\vec{b}-(\vec{a}.\vec{b}).\vec{c}$

$\Rightarrow (\vec{a}.\vec{b}).\vec{c}=(\vec{b}.\vec{c})\vec{a} \Rightarrow \vec{a}\parallel\vec{c}$.

44. (a) $\overrightarrow{CA}=(2-a)\hat{i}+2\hat{j}$;

$\overrightarrow{CB}=(1-a)\hat{i}-6\hat{k}$

$\overrightarrow{CA}.\overrightarrow{CB}=0 \Rightarrow (2-a)(1-a)=0$

$\Rightarrow$ a = 2, 1

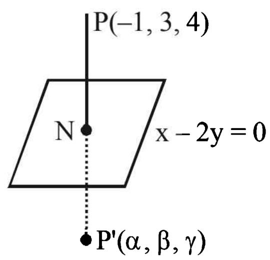
45. (a) Equation of lines $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$

$\frac{x-b'}{a'}=\frac{y}{1}=\frac{z-d'}{c'}$

Line are perpendicular $\Rightarrow aa'+1+cc'=0$

46. (d) Eq^n of PN:-

$\frac{x+1}{1}=\frac{y-3}{-2}=\frac{z-4}{0}=\lambda$



N $(\lambda-1, -2\lambda+3-4)$

It lies on $x-2y=0$

$\Rightarrow \quad \lambda-1+4\lambda-6=0$

$\Rightarrow \quad \lambda=7/5$

$N\left(\frac{2}{5},\frac{1}{5},4\right)$

N is mid point of PP'

$\therefore\ \alpha-1=\frac{4}{5},\beta+3=\frac{2}{5},r+4=8$

$\Rightarrow\ \alpha=\frac{9}{5},\beta=\frac{-13}{5},r=4$

$\therefore$ Image is $\left(\frac{9}{5},\frac{-13}{5},4\right)$

47. (b) Let the angle of line makes with the positive direction of z-axis is α direction cosines of line with the +ve directions of x-axis, y-axis, and z-axis is l, m, n respectively.

$\therefore\ l=\cos\frac{\pi}{4}, m=\cos\frac{\pi}{4}, n=\cos\alpha$

as we know that, $l^2+m^2+n^2=1$

$\therefore\ \cos^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}+\cos^2\alpha=1$

$\Rightarrow\ \frac{1}{2}+\frac{1}{2}+\cos^2\alpha=1$

$\Rightarrow\ \cos^2\alpha=0 \Rightarrow\ \alpha=\frac{\pi}{2}$

Hence, angle with positive direction of the z-axis is $\frac{\pi}{2}$

48. (b) Given $|2\hat{u}\times 3\hat{v}|=1$ and θ is acute angle between $\hat{u}$ and $\hat{v}$, $|\hat{u}|=1, |\hat{v}|=1 \Rightarrow 6|\hat{u}||\hat{v}||\sin\theta|=1$

$\Rightarrow\ 6|\sin\theta|=1 \Rightarrow\ \sin\theta=\frac{1}{6}$

Hence, there is exactly one value of θ for which $2\hat{u}\times 3\hat{v}$ is a unit vector.

49. (c) For given sphere centre is (3, 6, 1)
Coordinates of one end of diameter of the sphere are (2, 3, 5). Let the coordinates of the other end of diameter are (α, β, γ)

$\therefore\ \frac{\alpha+2}{2}=3, \frac{\beta+3}{2}=6, \frac{\gamma+5}{2}=1$

$\Rightarrow\ \alpha=4, \beta=9$ and $\gamma=-3$

$\therefore$ Coordinate of other end of diameter are (4, 9, –3)

50. (b) Given $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{c}=x\hat{i}+(x-2)\hat{j}-\hat{k}$

If $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $[\vec{a}\ \vec{b}\ \vec{c}]=0$

i.e. $\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 2\\ x & (x-2) & -1\end{vmatrix}=0$

$\Rightarrow\ 1[1-2(x-2)]-1[-1-2x]+1[x-2+x]=0$
$\Rightarrow\ 1-2x+4+1+2x+2x-2=0$
$\Rightarrow\ 2x=-4 \Rightarrow\ x=-2$

51. (c) Let the direction cosines of line L be l, m, n, then
$2l+3m+n=0$ ....(i)
and $l+3m+2n=0$ ....(ii)
on solving equation (i) and (ii), we get

$\frac{l}{6-3}=\frac{m}{1-4}=\frac{n}{6-3} \Rightarrow \frac{l}{3}=\frac{m}{-3}=\frac{n}{3}$

Now $\frac{l}{3}=\frac{m}{-3}=\frac{n}{3}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{3^2+(-3)^2+3^2}}$

$\because\ l^2+m^2+n^2=1$

$\therefore\ \frac{l}{3}=\frac{m}{-3}=\frac{n}{3}=\frac{1}{\sqrt{27}}$

$\Rightarrow\ l=\frac{3}{\sqrt{27}}=\frac{1}{\sqrt{3}}, m=-\frac{1}{\sqrt{3}}, n=\frac{1}{\sqrt{3}}$

Line L, makes an angle α with +ve x-axis

$\therefore\ l=\cos\alpha \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

52. (d) $\because\ \vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$

$\therefore\ \vec{a}=\vec{b}+\lambda\vec{c}$

$\Rightarrow \alpha\hat{i}+2\hat{j}+\beta\hat{k}=\hat{i}+\hat{j}+\lambda(\hat{j}+\hat{k})$

$\Rightarrow \alpha=1, 2=1+\lambda, \beta=\lambda \Rightarrow \alpha=1, \beta=1$

53. (d) Clearly $\vec{a}=-\frac{8}{7}\vec{c}$

$\Rightarrow \vec{a}\,||\,\vec{c}$ and are opposite in direction

$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π.

54. (c) Equation of line through $(5, 1, a)$ and $(3, b, 1)$ is $\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$

$\therefore$ Any point on this line is a
$[-2\lambda+5, (b-1)\lambda+1, (1-a)\lambda+a]$
It crosses yz plane where $-2\lambda+5=0$

$\lambda=\frac{5}{2}\quad \therefore \left(0, (b-1)\frac{5}{2}+1, (1-a)\frac{5}{2}+a\right)=\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

$\Rightarrow (b-1)\frac{5}{2}+1=\frac{17}{2}$ and $(1-a)\frac{5}{2}+a=-\frac{13}{2}$

$\Rightarrow b=4$ and $a=6$

55. (a) The two lines intersect if shortest distance between them is zero i.e.

$\frac{(\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2}{|\vec{b}_1\times\vec{b}_2|}=0 \Rightarrow (\vec{a}_2-\vec{a}_1)\cdot\vec{b}_1\times\vec{b}_2=0$

where $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}_1=k\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2=2\hat{i}+3\hat{j}+\hat{k}, \vec{b}_2=3\hat{i}+k\hat{j}+2\hat{k}$

$\Rightarrow \begin{vmatrix}1 & 1 & -2\\ k & 2 & 3\\ 3 & k & 2\end{vmatrix}=0$

$\Rightarrow 1(4-3k)-1(2k-9)-2(k^2-6)=0$

$\Rightarrow -2k^2-5k+25=0 \Rightarrow k=-5$ or $\frac{5}{2}$

$\because k$ is an integer, therefore $k=-5$

56. (a) $\because$ The line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x+3y-\alpha z+\beta=0$

$\therefore$ Pt $(2, 1, -2)$ lies on the plane
i.e. $2+3+2\alpha+\beta=0$ or $2\alpha+\beta+5=0$ ....(i)
Also normal to plane will be perpendicular to line,

$\therefore\ 3\times1-5\times3+2\times(-\alpha)=0 \Rightarrow \alpha=-6$

From equation (i) then, $\beta=7$

$\therefore\ (\alpha, \beta)=(-6, 7)$

57. (b) Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be the initial and final points of the vector whose projections on the three coordiante axes are 6, – 3, 2
then
$x_2-x_1=6;\ y_2-y_1=-3;\ z_2-z_1=2$

So that direction ratios of $\overrightarrow{PQ}$ are 6, – 3, 2

$\therefore$ Direction cosines of $\overrightarrow{PQ}$ are

$$\frac{6}{\sqrt{6^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{6^2+(-3)^2+2^2}}, \frac{2}{\sqrt{6^2+(-3)^2+2^2}} = \frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$$

58. (d) $[3\vec{u}\ p\vec{v}\ p\vec{\omega}]-[p\vec{v}\ \vec{\omega}\ q\vec{u}]-[2\vec{\omega}\ q\vec{v}\ q\vec{u}]=0$

$\Rightarrow (3p^2-pq+2q^2)[\vec{u}\ \vec{v}\ \vec{\omega}]=0$

$\Rightarrow 3p^2-pq+2q^2=0 \quad (\because [\vec{u}\ \vec{v}\ \vec{\omega}]\neq 0)$

$\Rightarrow 2p^2+p^2-pq+\frac{q^2}{4}+\frac{7q^2}{4}=0$

$\Rightarrow 2p^2+\left(p-\frac{q}{2}\right)^2+\frac{7}{4}q^2=0$

$\Rightarrow p=0, q=0,\ p=\frac{q}{2} \Rightarrow p=0, q=0$

$\therefore$ Exactly one value of (p, q)

59. (d) $\vec{c}=\vec{b}\times\vec{a} \Rightarrow \vec{b}.\vec{c}=\vec{b}.(\vec{b}\times\vec{a}) \Rightarrow \vec{b}.\vec{c}=0$

$\Rightarrow \left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right).\left(\hat{i}-\hat{j}-\hat{k}\right)=0,$

where $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$b_1-b_2-b_3=0$ ...(i)

and $\vec{a}.\vec{b}=3 \Rightarrow (\hat{j}-\hat{k}).(b_1\hat{i}+b_2\hat{j}+b_3\hat{k})=3$

$\Rightarrow b_2-b_3=3$

From equation (i)

$b_1=b_2+b_3=(3+b_3)+b_3=3+2b_3$

$\vec{b}=(3+2b_3)\hat{i}+(3+b_3)\hat{j}+b_3\hat{k}$

From the option given, it is clear that b_3 equal to either 2 or -2.

If $b_3=2$ then $\vec{b}=7\hat{i}+5\hat{j}+2\hat{k}$ which is not possible

If $b_3=-2$, then $\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

60. (d) Since, $\vec{a},\vec{b}$ and $\vec{c}$ are mutually orthogonal

$\therefore \vec{a}.\vec{b}=0,\ \vec{b}.\vec{c}=0,\ \vec{c}.\vec{a}=0$

$\Rightarrow 2\lambda+4+\mu=0$ ...(i)

$\lambda-1+2\mu=0$ ...(ii)

On solving (i) and (ii), we get $\lambda=-3, \mu=2$

61. (a) A(3, 1, 6); B = (1, 3, 4)
Mid-point of AB = (2, 2, 5) lies on the plane.
and d.r's of AB = (2, –2, 2)
d.r's of normal to plane = (1, –1, 1).
Direction ratio of AB and normal to the plane are proportional therefore,
AB is perpendicular to the plane
$\therefore$ A is image of B
Statement-2 is correct but it is not correct explanation.

62. (b) Direction cosines of the line :

$\ell=\cos 45°=\frac{1}{\sqrt{2}},\ m=\cos 120°=\frac{-1}{2},\ n=\cos\theta$

where θ is the angle, which line makes with positive z-axis.

Now $\ell^2+m^2+n^2=1$

$\Rightarrow \frac{1}{2}+\frac{1}{4}+\cos^2\theta=1,\ \cos^2\theta=\frac{1}{4}$

$\Rightarrow \cos\theta=\frac{1}{2}$ (θ being acute) $\Rightarrow \theta=\frac{\pi}{3}$

63. (d) If θ be the angle between the given line and plane, then

$$\sin\theta=\frac{1\times1+2\times2+\lambda\times3}{\sqrt{1^2+2^2+\lambda^2}.\sqrt{1^2+2^2+3^2}}=\frac{5+3\lambda}{\sqrt{14}.\sqrt{5+\lambda^2}}$$

But it is given that $\theta=\cos^{-1}\sqrt{\frac{5}{14}} \Rightarrow \sin\theta=\frac{3}{\sqrt{14}}$

$\therefore \frac{5+3\lambda}{\sqrt{14}\sqrt{5+\lambda^2}}=\frac{3}{\sqrt{14}} \Rightarrow \lambda=\frac{2}{3}$

64. (d) We have $\vec{a}\cdot\vec{b}=0,\ \vec{a}\cdot\vec{a}=1,\ \vec{b}\cdot\vec{b}=1$

$(2\vec{a}-\vec{b})\cdot[(\vec{a}\times\vec{b})\times(\vec{a}+2\vec{b})]$

$=(2\vec{a}-\vec{b})\cdot[\{\vec{a}\cdot(\vec{a}+2\vec{b})\}\vec{b}-\{\vec{b}\cdot(\vec{a}+2\vec{b})\vec{a}\}]$

$=(2\vec{a}-\vec{b})\cdot[(\vec{a}\cdot\vec{a}+2\vec{a}\cdot\vec{b})\vec{b}-(\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{b})\vec{a}]$

$=(2\vec{a}-\vec{b})\cdot[\vec{b}-2\vec{a}]=4\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{b}-4\vec{a}\cdot\vec{a}=-5$

65. (c) $\vec{a}.\vec{b}\neq 0,\ \vec{a}.\vec{d}=0$

Now, $\vec{b}\times\vec{c}=\vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}\times\vec{c})=\vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=(\vec{a}.\vec{d})\vec{b}-(\vec{a}.\vec{b})\vec{d}$

$\Rightarrow (\vec{a}.\vec{b})\vec{d}=-(\vec{a}.\vec{c})\vec{b}+(\vec{a}.\vec{b})\vec{c}$

$\vec{d}=\vec{c}-\left(\frac{\vec{a}.\vec{c}}{\vec{a}.\vec{b}}\right)\vec{b}$

66. (a) The direction ratios of the line segment joining points A(1, 0, 7) and B(1, 6, 3) are 0, 6, –4.
The direction ratios of the given line are 1, 2, 3.
Clearly $1\times0+2\times6+3\times(-4)=0$
So, the given line is perpendicular to line AB.
Also, the mid point of A and B is (1, 3, 5) which lies on the given line.
So, the image of B in the given line is A, because the given line is the perpendicular bisector of line segment joining points A and B, But statement-2 is not a correct explanation for statement-1.

67. (c) Let $\vec{c}=\hat{a}+2\hat{b}$ and $\vec{d}=5\hat{a}-4\hat{b}$

Since $\vec{c}$ and $\vec{d}$ are perpendicular to each other

$\therefore \vec{c}.\vec{d}=0 \Rightarrow \left(\hat{a}+2\hat{b}\right).\left(5\hat{a}-4\hat{b}\right)=0$

$\Rightarrow 5+6\hat{a}.\hat{b}-8=0 \quad (\because \hat{a}.\hat{a}=1)$

$\Rightarrow \hat{a}.\hat{b}=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{3}$

68. (a) Given equation of a plane is $x-2y+2z-5=0$
So, Equation of parallel plane is given by $x-2y+2z+d=0$

Now, it is given that distance from origin to the parallel plane is 1.

$$\therefore \left|\frac{d}{\sqrt{1^2+2^2+2^2}}\right| = 1 \Rightarrow d = \pm 3$$

So equation of required plane $x - 2y + 2z \pm 3 = 0$

69. (c) Given lines in vector form are

$$\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(2\vec{i} + 3\vec{j} + 4\vec{j})$$

and $\vec{r} = (3\hat{i} + k\hat{j}) + \mu(\hat{i} + 2\hat{j} + \hat{k})$

These will intersect if shortest distance between them $= 0$

i.e. $(\vec{a}_2 - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2 = 0$

$$\Rightarrow \begin{vmatrix} 3-1 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

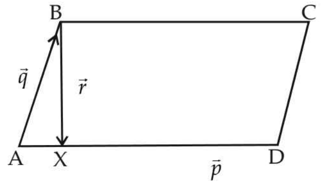
$\Rightarrow 2(-5) - (k+1)(-2) - 1(1) = 0$

$\Rightarrow k = 9/2$

70. (b) Let $ABCD$ be a parallelogram such that

$\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle.

We have



$$\overrightarrow{AX} = \left(\frac{\vec{p}\cdot\vec{q}}{|\vec{p}|}\right)\left(\frac{\vec{p}}{|\vec{p}|}\right) = \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|^2}\vec{p}$$

Let $\vec{r} = \overrightarrow{BX} = \overrightarrow{BA} + \overrightarrow{AX} = -\vec{q} + \dfrac{\vec{p}\cdot\vec{q}}{|\vec{p}|^2}\vec{p}$

71. (c) $2x + y + 2z - 8 = 0$ ...(Plane 1)

$2x + y + 2z + \dfrac{5}{2} = 0$ ...(Plane 2)

Distance between Plane 1 and 2

$$= \left|\frac{-8 - \frac{5}{2}}{\sqrt{2^2+1^2+2^2}}\right| = \left|\frac{-21}{6}\right| = \frac{7}{2}$$

72. (c) Given lines will be coplanar

$$\text{If} \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow -1(1+2k) - (1+k^2) + 1(2-k) = 0$

$\Rightarrow k = 0, -3$

73. (c) $\because$ M is mid point of BC

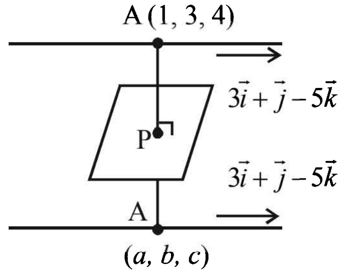
$$\therefore \overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$$

$= 4\vec{i} + \vec{j} + 4\vec{k}$

Length of median AM

$= \sqrt{16+1+16} = \sqrt{33}$

74. (c)



$$\frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = \lambda(\text{let})$$

$\Rightarrow a = 2\lambda + 1$

$b = 3 - \lambda$

$c = 4 + \lambda$

$$P = \left(\frac{a+1}{2}, \frac{b+3}{2}, \frac{c+4}{2}\right)$$

$$= \left(\lambda+1, \frac{6-\lambda}{2}, \frac{\lambda+8}{2}\right)$$

$$\therefore 2(\lambda+1) - \frac{6-\lambda}{2} + \frac{\lambda+8}{2} + 3 = 0$$

$3\lambda + 6 = 0 \Rightarrow \lambda = -2$

$a = -3, b = 5, c = 2$

Required line is $\dfrac{x+3}{3} = \dfrac{y-5}{1} = \dfrac{z-2}{-5}$

75. (c) Given

$l + m + n = 0$ and $l^2 = m^2 + n^2$

Now, $(-m-n)^2 = m^2 + n^2$

$\Rightarrow mn = 0 \Rightarrow m = 0$ or $n = 0$

If $m = 0$ then $l = -n$

We know $l^2 + m^2 + n^2 = 1 \Rightarrow n = \pm\dfrac{1}{\sqrt{2}}$

i.e. $(l_1, m_1, n_1) = \left(-\dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}}\right)$

If $n = 0$ then $l = -m$

$l^2 + m^2 + n^2 = 1 \Rightarrow 2m^2 = 1$

$\Rightarrow m = \pm\dfrac{1}{\sqrt{2}}$

Let $m = \dfrac{1}{\sqrt{2}} \Rightarrow l = -\dfrac{1}{\sqrt{2}}$ and $n = 0$

$(l_2, m_2, n_2) = \left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0\right)$

$\therefore \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

76. **(b)** L.H.S $= (\vec{a}\times\vec{b}).[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})]$

$= (\vec{a}\times\vec{b}).[(\vec{b}\times\vec{c}.\vec{a})\vec{c}-(\vec{b}\times\vec{c}.\vec{c})\vec{a}]$

$= (\vec{a}\times\vec{b}).[[\vec{b}\,\vec{c}\,\vec{a}]\vec{c}] \qquad [\because \vec{b}\times\vec{c}.\vec{c}=0]$

$= [\vec{a}\,\vec{b}\,\vec{c}].(\vec{a}\times\vec{b}.\vec{c})=[\vec{a}\,\vec{b}\,\vec{c}]^2$

$[\vec{a}\times\vec{b}\quad \vec{b}\times\vec{c}\quad \vec{c}\times\vec{a}]=[\vec{a}\,\vec{b}\,\vec{c}]^2$

So $\lambda = 1$

77. **(c)** $(\vec{a}\times\vec{b})\times\vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a} \Rightarrow -\vec{c}\times(\vec{a}\times\vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -(\vec{c}.\vec{b})\vec{a}+(\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow -|\vec{b}||\vec{c}|\cos\theta\vec{a}+(\vec{c}.\vec{a})\vec{b} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\because$ $\vec{a},\vec{b},\vec{c}$ are non collinear, the above equation is possible only when

$-\cos\theta = \frac{1}{3}$ and $\vec{c}.\vec{a} = 0$

$\Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$; $\theta \in$ II quad

78. **(a)** Equation of the plane containing the lines

$2x-5y+z=3$ and $x+y+4z=5$ is

$2x-5y+z-3+\lambda(x+y+4z-5)=0$

$\Rightarrow (2+\lambda)x+(-5+\lambda)y+(1+4\lambda)z+(-3-5\lambda)=0$ ...(i)

Since the plane (i) parallel to the given plane $x+3y+6z=1$

$\therefore \frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6} \Rightarrow \lambda = -\frac{11}{2}$

Hence equation of the required plane is

$\left(2-\frac{11}{2}\right)x+\left(-5-\frac{11}{2}\right)y+\left(1-\frac{44}{2}\right)z+\left(-3+\frac{55}{2}\right)=0$

$\Rightarrow \quad x+3y+6z=7$

79. **(b)** General point on given line $\equiv P(3r+2, 4r-1, 12r+2)$

Point P must satisfy equation of plane

$(3r+2)-(4r-1)+(12r+2)=16$

$11r+5=16$

$r=1$

$P(3\times 1+2, 4\times 1-1, 12\times 1+2)=P(5, 3, 14)$

distance between P and (1, 0, 2)

$D=\sqrt{(5-1)^2+3^2+(14-2)^2}=13$

80. **(b)** Line lies in the plane $\Rightarrow$ (3, –2, –4) lie in the plane

$\Rightarrow 3\ell-2m+4=9$ or $3\ell-2m=5$ ..... (1)

Also, ℓ, m, –1 are dr's of line perpendicular to plane and 2, –1, 3 are dr's of line lying in the plane

$\Rightarrow 2\ell-m-3=0$ or $2\ell-m=3$ .....(2)

Solving (1) and (2) we get $\ell=1$ and $m=-1$

$\Rightarrow \ell^2+m^2=2.$

81. **(b)** $\vec{a}\times(\vec{b}\times\vec{c})=\frac{\sqrt{3}}{2}(\vec{b}+\vec{c})$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\frac{\sqrt{3}}{2}\vec{b}+\frac{\sqrt{3}}{2}\vec{c}$

On comparing both sides

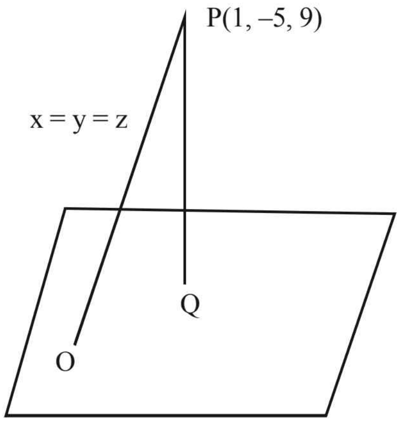
$\vec{a}\cdot\vec{b}=-\frac{\sqrt{3}}{2} \Rightarrow \cos\theta=-\frac{\sqrt{3}}{2}$

$[\because \vec{a}$ and $\vec{b}$ are unit vectors]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\theta = \frac{5\pi}{6}$

82. **(d)**



eq^n of PO : $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

$\Rightarrow x=\lambda+1;\ y=\lambda-5; z=\lambda+9.$

Putting these in eq^n of plane :-

$\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow \lambda=-10$

$\Rightarrow$ O is (–9, –15, –1)

$\Rightarrow$ distance OP $=10\sqrt{3}$.